Locally Covert Learning

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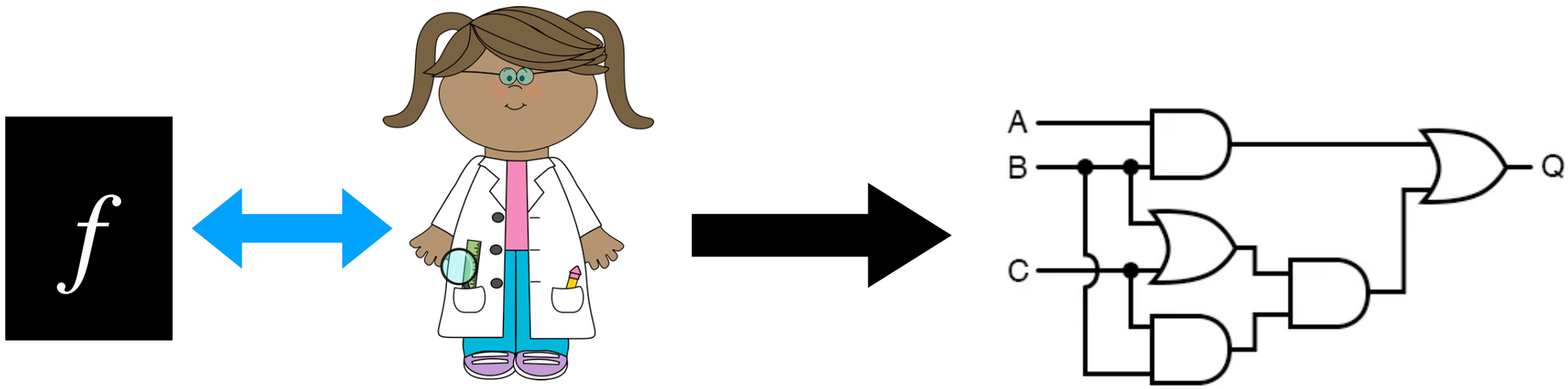
Learning Boolean Functions



1. Polynomial-time learner gets *access to a function* $f : \{0,1\}^n \to \{0,1\}$

2. Learner's goal is to output a function h that agrees with f on most inputs.

The Learning Model, Part I

By "$\mathscr{H}$ is learnable", we mean there exists a learning algorithm that is:

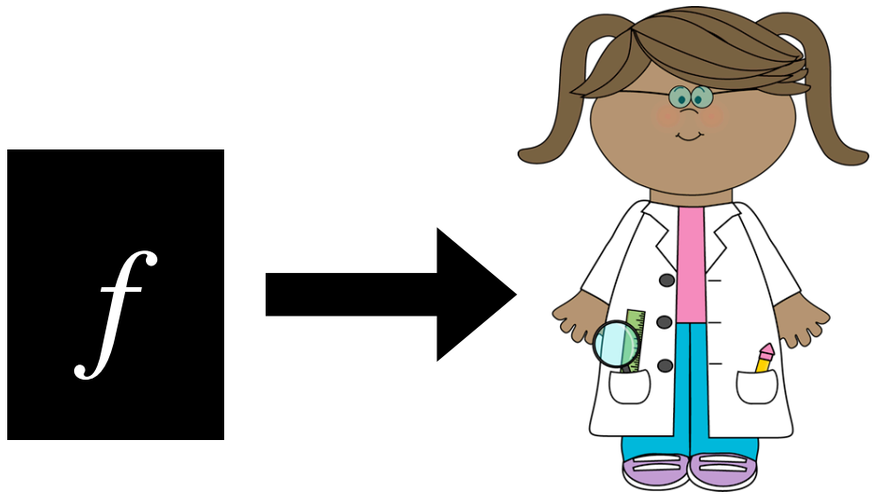
Efficient:
Given accuracy parameter α and confidence parameter δ, learner runs in time poly$(n,\ 1/\alpha,\ \log(1/\delta))$.

(**Weakly) Agnostic:**
If target function f is ϵ-close to some $h^\star \in \mathscr{H}$,
the learner outputs a hypothesis h that is $(O(\epsilon) + \alpha)$-close to f,
with all but δ probability.

Distribution-Specific: "closeness" is measured wrt the uniform distribution

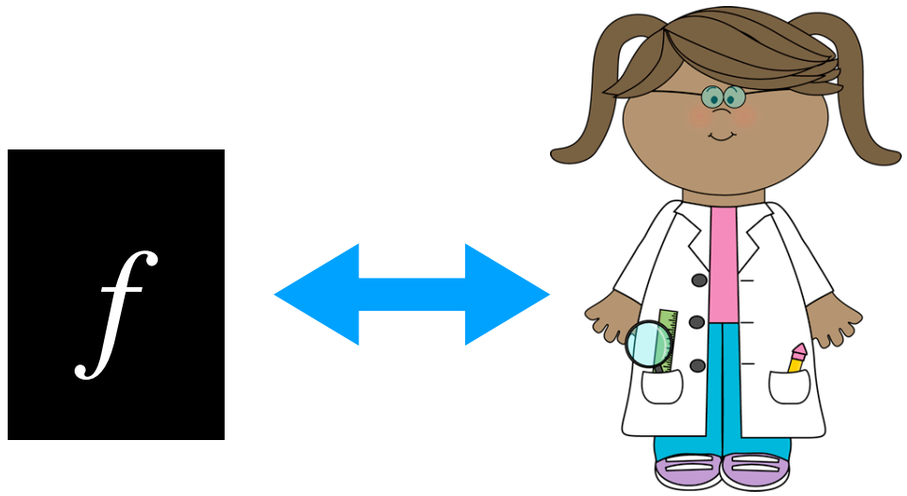
Improper: Learner can output any circuit, not necessarily in $\mathscr{H}$.

The Learning Model, Part II
Types of Function Access



Passive Learning:

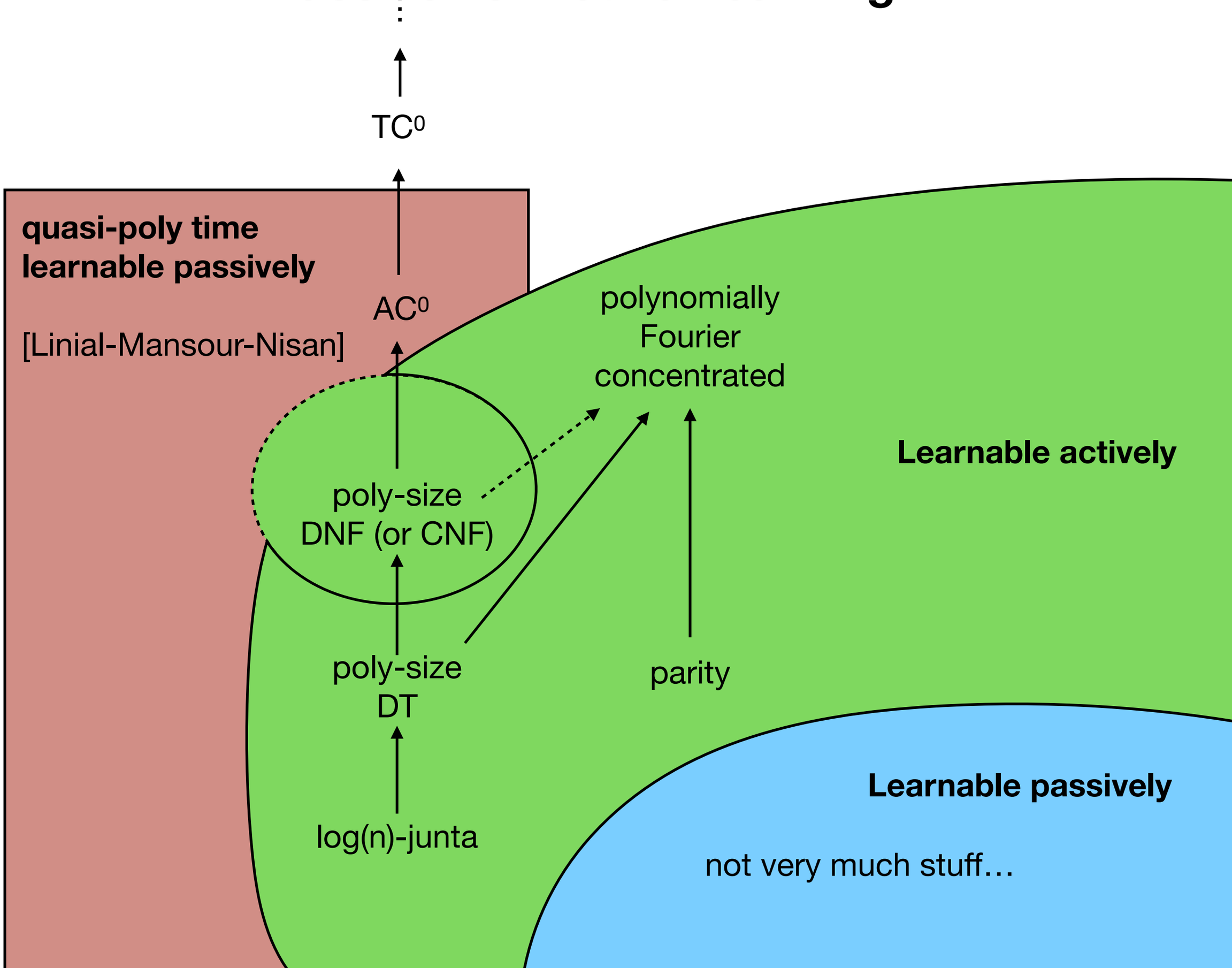
Learner gets pairs $(x, f(x))$ for uniformly random x



Active Learning:

Learner gets oracle access to f.

Passive vs. Active Learning



where’s the adversary?

Adversarial Learning

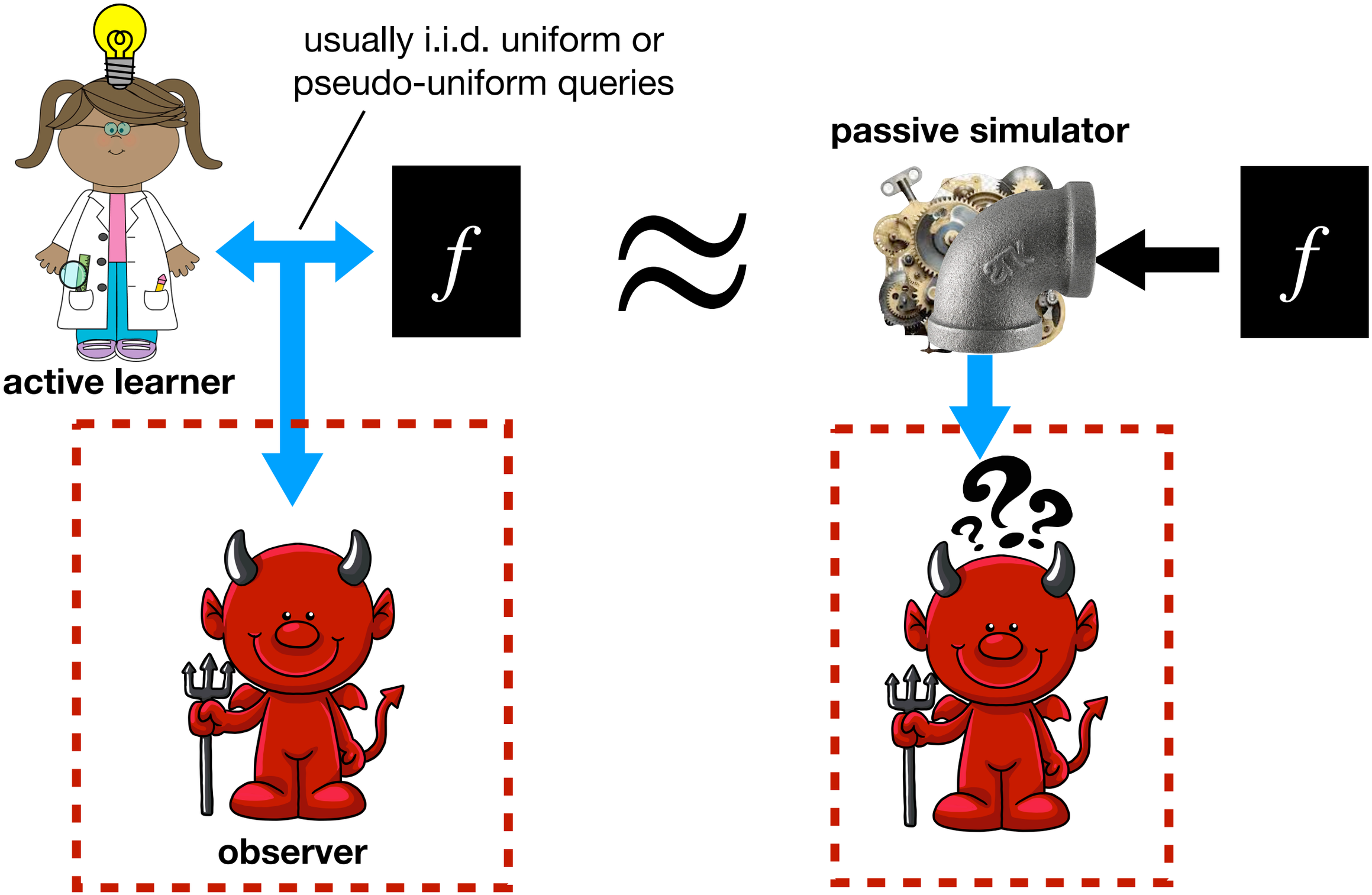
By now a burgeoning field. Includes, but not limited to:

1. **Covert Learning**: [Canetti-Karchmer ’21, IKOS ’19]
   curious eavesdropper tries to piggyback on the queries of an active learner.

2. **Verifiable Learning:** [Goldwasser-Rothblum-Shafer-Yehudayoff ’20]
   untrusted prover claims that a hypothesis h approximates f near-optimally (compared to some class of functions).

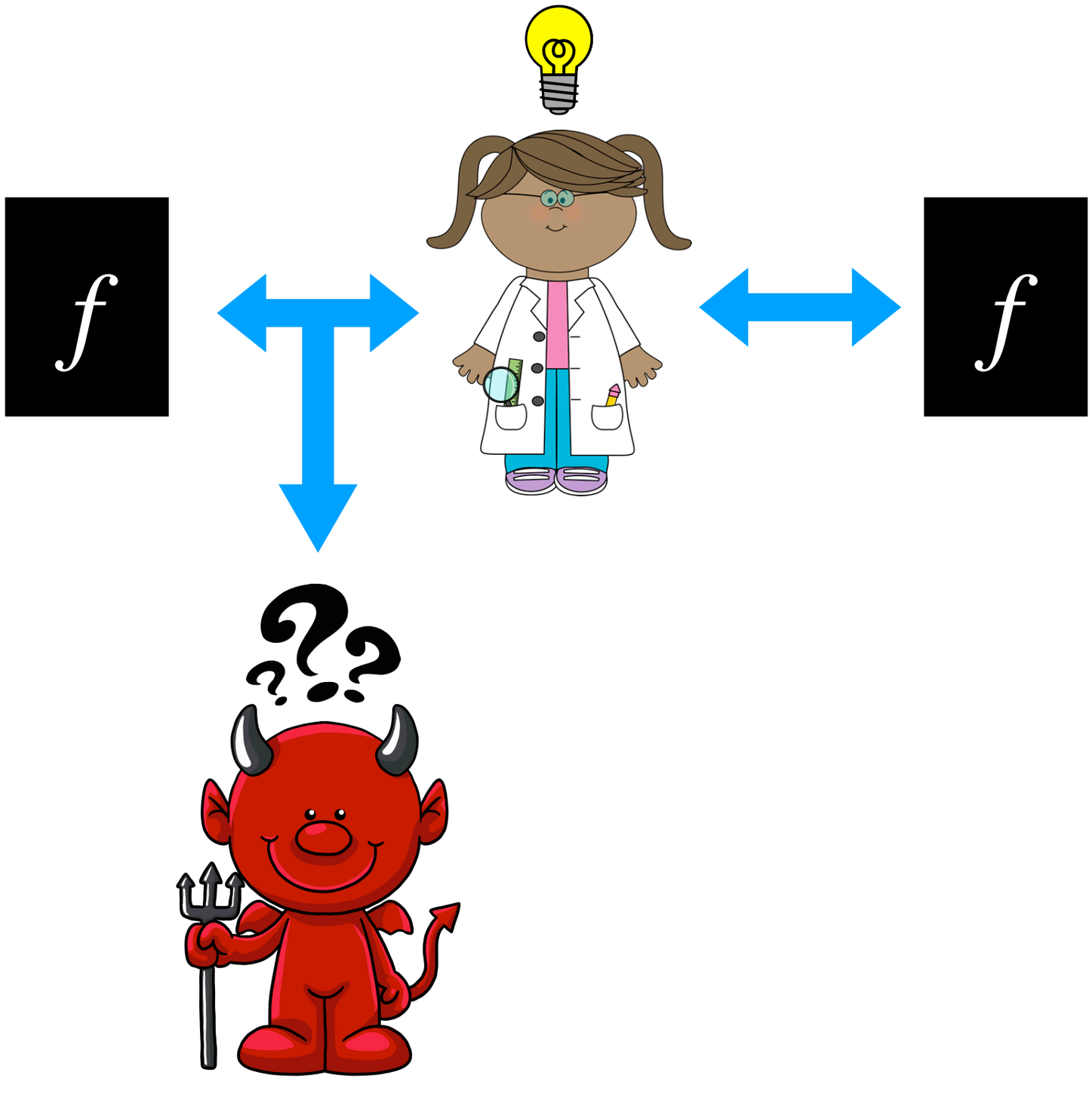
Covert Learning

[Canetti-Karchmer ’21, IKOS ’19]



(1-of-2) Locally Covert Learning

[IKOS19]



Why Study Covert Learning?
Scenario 1: Delegating Scientific Discovery

[Canetti-Karchmer '21]

Plan: Learn a function f for which:
random examples are cheap / useless
queries are relatively useful but expensive,

For example, an organism's genome → phenome map

Problem: Want to delegate to specialists, but …
what if they sell resulting data to your competitors?

Solution: use covert learning $\Longrightarrow$ their data has no resale value

Scenario 2: Verifiable Learning

Same Plan: Delegate the query-learning of a function f; assume cheap but useless random examples for f.

New Problem [Goldwasser-Rothblum-Shafer-Yehudayoff '21]**:** How to ensure we receive a near-optimal circuit?

One Approach: Tell learner what queries to make (following a covert learning algorithm). Hide "test queries" (using random examples)

➡ If test queries are correct, most others must be as well.

➡ If learning algorithm is also "robust" then a few incorrect query answers can't ruin the output.

Scenario 3: Model Extraction

[Canetti-Karchmer ’21]

Plan: Sell AI as a service (e.g. chat GPT)

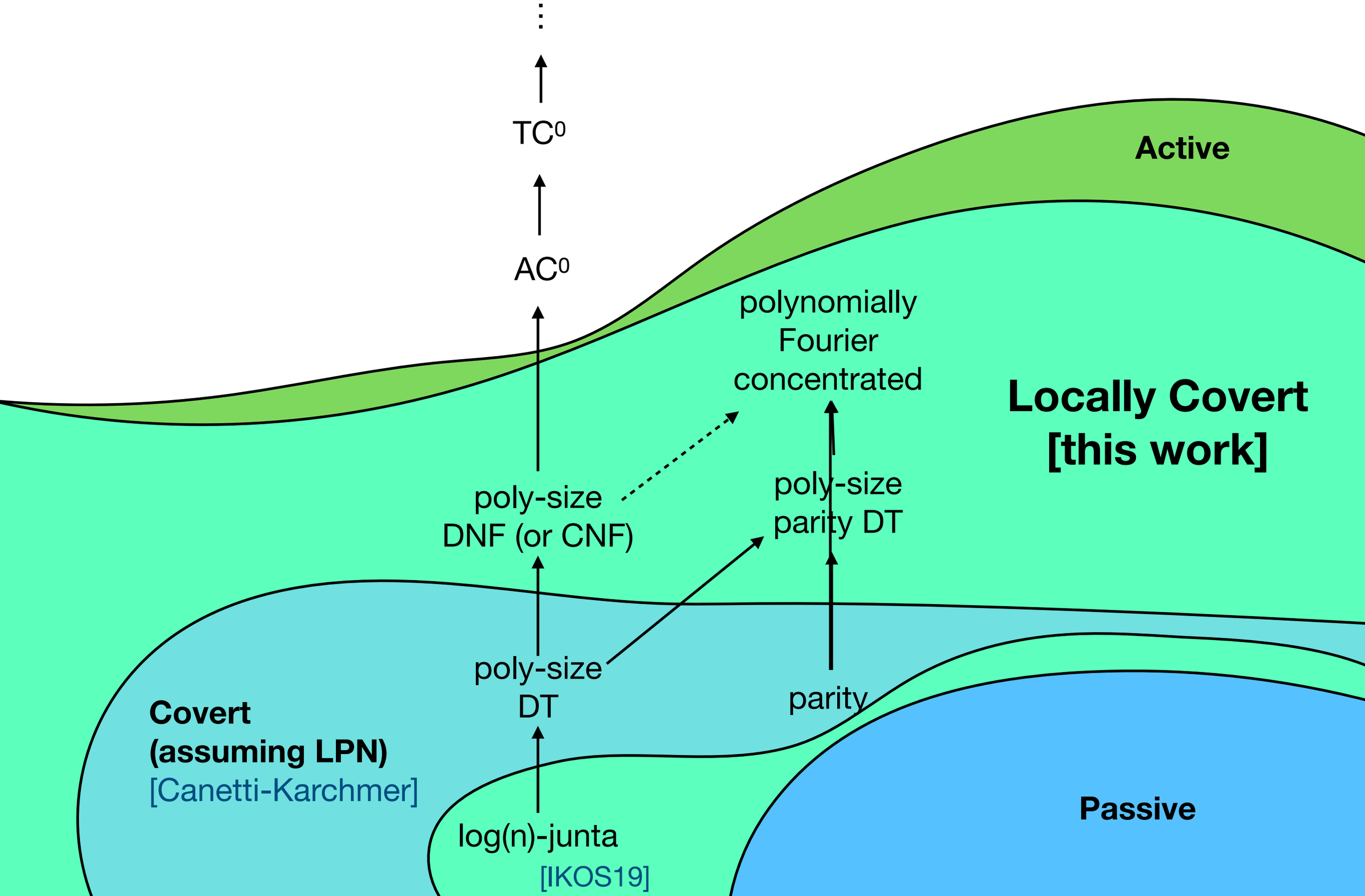
- Generally trained on random data (more scalable)

Problem: Can competitor use queries to clone the model?

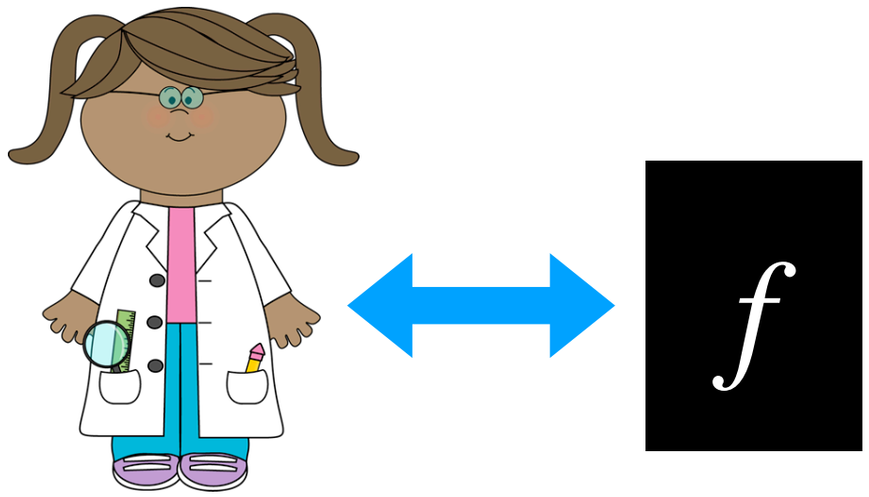
Defense?? Block users who make weird query patterns

😢 Can’t really work against a covert learner

Passive ⊆ Covert ⊆ Active



The Goldreich-Levin Theorem



Learning Version:

Given oracle access to f, one can efficiently find all *parity functions* γ that are even weakly correlated with f

Crypto Version:

Let g be a OWF. Then $\langle \mathbf{x}, \mathbf{r} \rangle \pmod 2$ is *hard-core* for $(g(\mathbf{x}), \mathbf{r})$.

Proof assuming Learning Version:

1. $\langle \mathbf{x}, \cdot \rangle \pmod 2$ is a parity function.
2. If not hard-core, then an adversary $\mathscr{A}(g(\mathbf{x}), \cdot)$ weakly predicts $\langle \mathbf{x}, \mathbf{r} \rangle$.
3. $\mathrm{GL}^{\mathscr{A}(g(\mathbf{x}),\cdot)}$ outputs a list containing $\mathbf{x} \Longrightarrow$ contradicts that g is a OWF.

Which "Goldreich-Levin Algorithm"?

Rackoff's Algorithm

- Uses derandomization (querying all subset sums of $\approx \log(n)$ random vectors in $\mathbb{F}_2^n$)

➡ Queries are not statistically uniform

The Original [Goldreich-Levin]

- Uses Fourier analysis
- Well-known in learning theory

This Work:

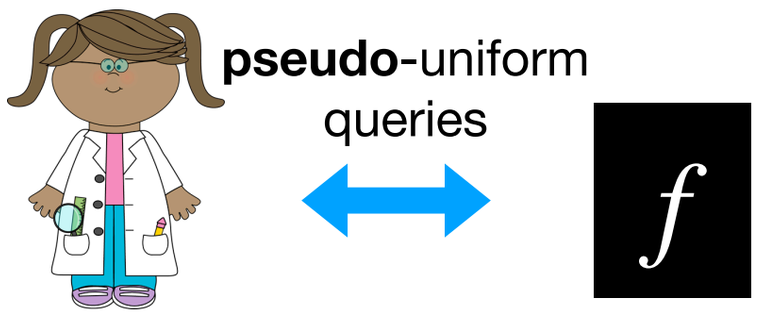
Original algorithm is basically already 1-out-of-2 covert.

Small modification gives $(k-1)$-out-of-k covertness.

(Locally) Covert Goldreich-Levin Algorithms

Previous Lemma: [Canetti-Karchmer] (following [Rackoff])

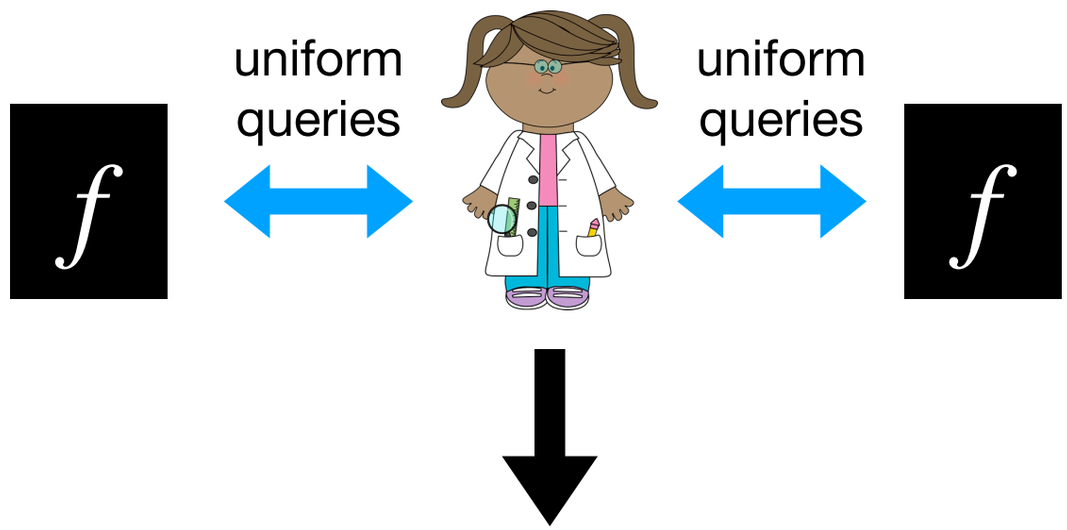
Assuming LPN is subexponentially hard, there is a **computationally** covert algorithm for **low-degree** Goldreich-Levin learning



all **log(n)-variable** γ s.t. $|\hat{f}(\gamma)| \geq \tau$
(except with δ probability)

Our Main Theorem (following [Goldreich-Levin]):

For any constant k, there is a **perfectly** $(k-1)$-out-of-k covert algorithm for Goldreich-Levin learning



all γ s.t. $|\hat{f}(\gamma)| \geq \tau$
(except with δ probability)

Fourier Analysis Essentials

For $\gamma \in \mathbb{F}_2^n$, let $\hat{f}(\gamma) \in [-1,1]$ denote the *correlation* of f with the parity function $\langle \gamma, \cdot \rangle$. Call $\hat{f}(\gamma)^2$ the *weight* of γ.

Fact: $\sum_{\gamma \in \mathbb{F}_2^n} \hat{f}(\gamma)^2 = 1.$

Not many heavy parities.

Not many heavy prefixes.

and 1-of-2 covertly

Lemma: With queries to f, one can efficiently estimate

$$\text{weight}(p) := \sum_{s \in \mathbb{F}_2^{n-k}} \hat{f}(p \circ s)^2 \text{ for } \textit{any "prefix"} \; p \in \mathbb{F}_2^k,$$

$\circ$: concatenation

will prove this later

Weighing Parity Prefixes $\Longrightarrow$ Goldreich-Levin

Basic Idea: Maintain a list of candidate *prefixes* of heavy parities γ (those with $\hat{f}(\gamma)^2 \geq \tau$), starting with 1-bit prefixes $\{0,1\}$.

1. Weigh each prefix in the list and throw away light prefixes (those with weight $< \tau$)

   ➡ At most $1/\tau$ prefixes.

2. Replace remaining prefixes p by $p \circ 0$ and $p \circ 1$

   ➡ At most $2/\tau$ prefixes

3. Repeat until prefixes are n-bit strings.

How To Weigh ~~Prefixes~~

Affine Spaces

Lemma:

and 1-of-2 covertly

With queries to f, one can efficiently estimate

$$\text{weight}(p) := \sum_{s \in \mathbb{F}_2^{n-k}} \hat{f}(p \circ s)^2 \text{ for } \textit{any "prefix"} \ p \in \mathbb{F}_2^k.$$

More Generally:

and 1-of-2 covertly

With queries to f, one can efficiently estimate

$$\text{weight}(A) := \sum_{\gamma \in A} \hat{f}(\gamma)^2 \text{ for } \textit{any affine subspace} \ A \subseteq \mathbb{F}_2^n$$

How To Weigh Affine Spaces

Lemma:

For *any affine subspace* $A \subseteq \mathbb{F}_2^n, \quad A = \gamma^\star + V,$

one can efficiently estimate $\text{weight}(A) := \sum_{\gamma \in A} \hat{f}(\gamma)^2$

1-of-2 covertly

Formula:

x_1 and x_2 individually uniformly random

$$\sum_{\gamma \in A} \hat{f}(\gamma)^2 = \mathop{\mathbb{E}}_{x_1 + x_2 \in V^\perp} \left[f(x_1) \cdot f(x_2) \cdot \gamma^\star(x_1 + x_2) \right]$$

Expectation can be directly empirically estimated

$(k-1)$-of-k Covertness

Previous formula naturally generalizes:

If $A = \gamma^\star + V$ is an affine subspace of $\mathbb{F}_2^n$, then

$$\sum_{\gamma \in A} \hat{f}(\gamma)^k = \mathop{\mathbb{E}}_{x_1+\cdots+x_k \in V^\perp} \left[f(x_1)\cdots f(x_k) \cdot \gamma^\star(x_1 + \cdots x_k) \right]$$

"k-weight of A"

$(k-1)$-wise uniform

- To get $(k-1)$-of-k covert GL, apply same strategy, using k-weight instead of 2-weight

Goldreich-Levin with k-weights (k even WLOG)

Strategy: Maintain a list of candidate *l-bit prefixes* of heavy parities γ (those with $\hat{f}(\gamma)^2 \geq \tau$), starting with $\{0,1\}$.

1. Weigh each prefix in the list and throw away light prefixes (those with k-weight $< \tau^{k/2}$)

   ➡ At most $1/\tau^{k/2}$ prefixes because 2-weight > k-weight

2. Replace remaining prefixes p by $p \circ 0$ and $p \circ 1$

   ➡ At most $2/\tau^{k/2}$ prefixes

   τ can be 1 / poly(n), so running time $n^{O(k)}$

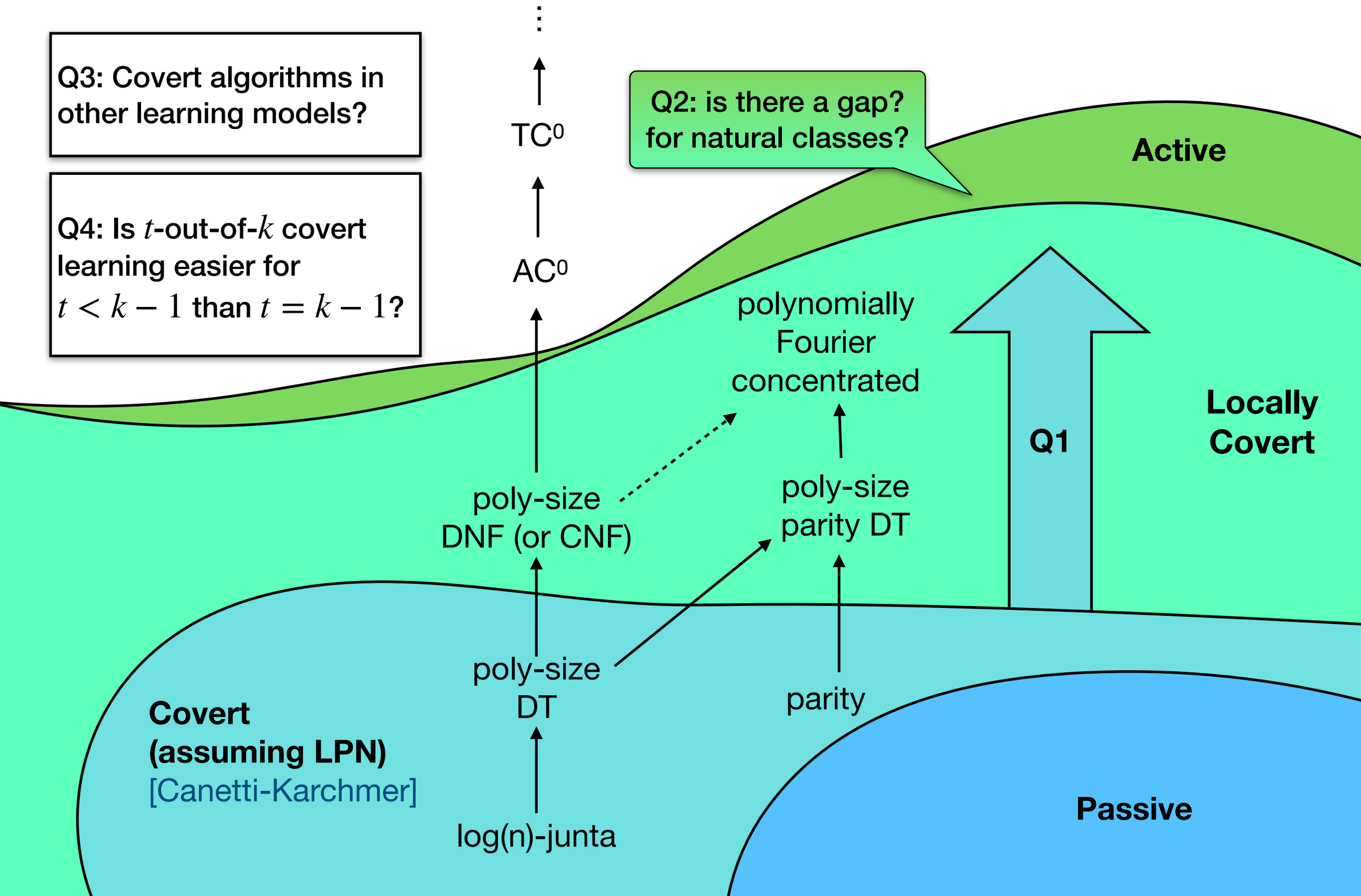
3. Repeat until prefixes are n-bit strings.

Future Work?

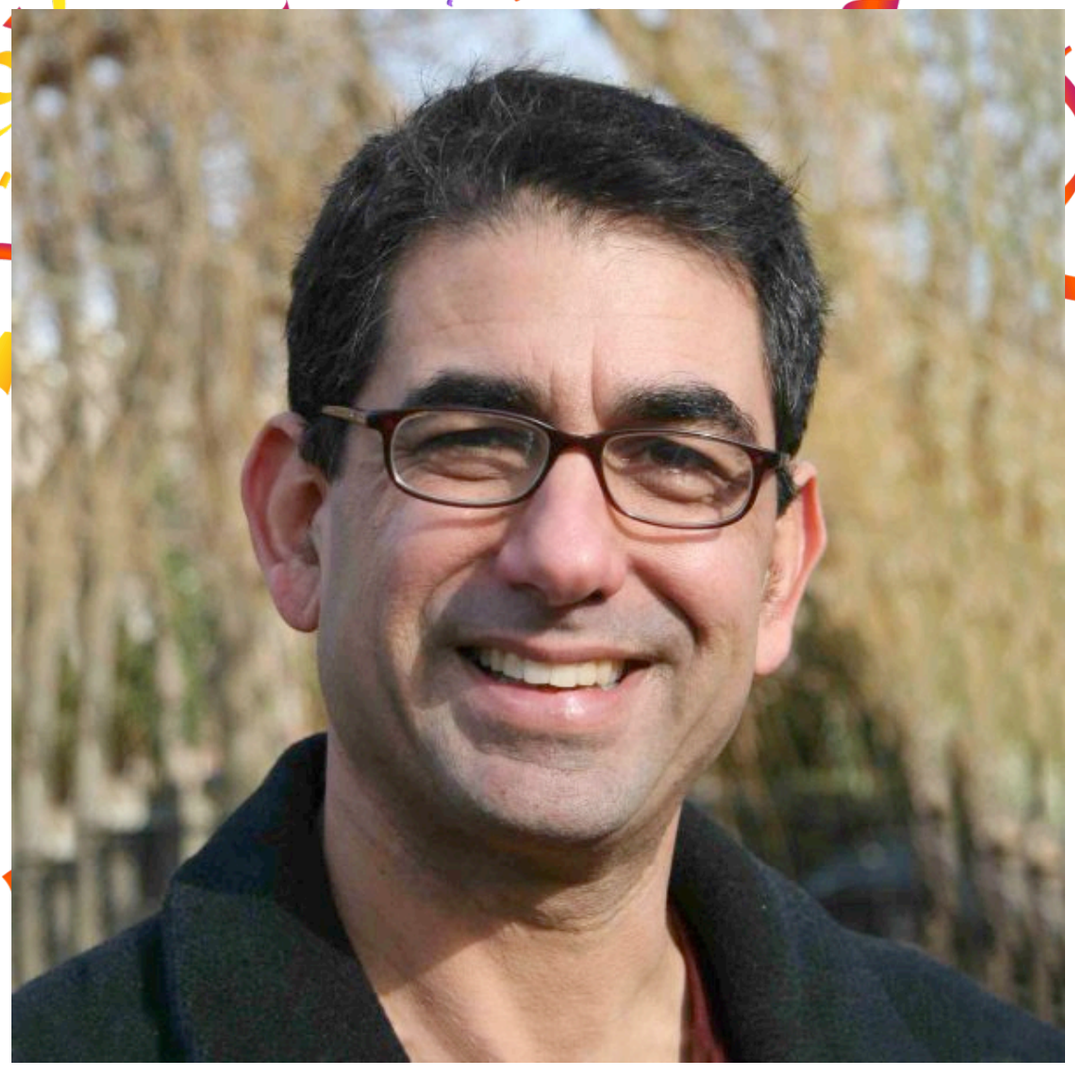
Q3: Covert algorithms in other learning models?

Q4: Is t-out-of-k covert learning easier for $t < k - 1$ than $t = k - 1$?

Q2: is there a gap? for natural classes?



Thanks & Happy Birthday!



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