Locally Covert Learning

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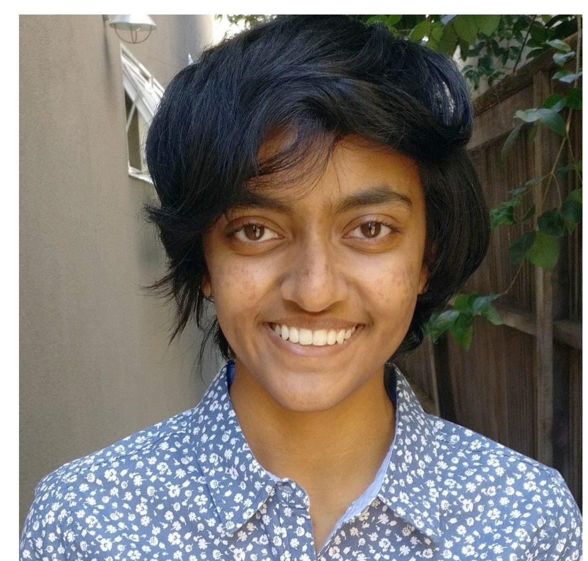
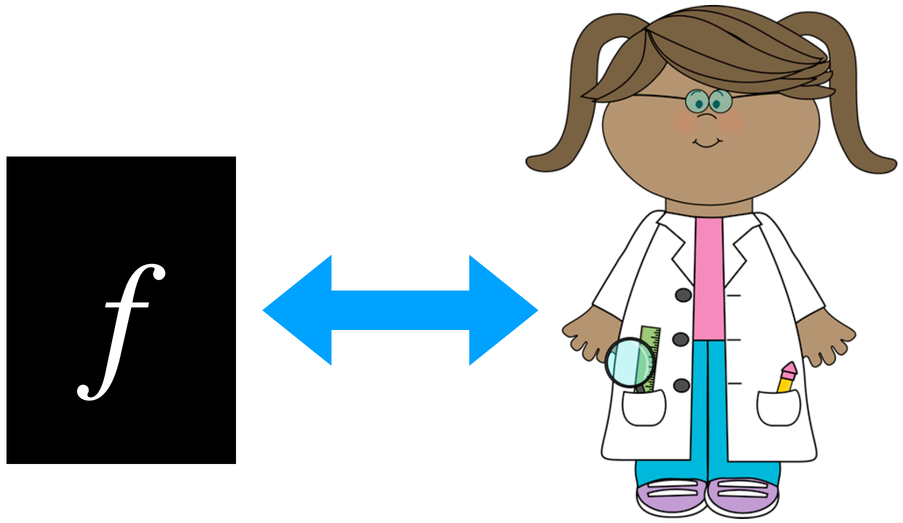
NTT Research

Ruta Jawale
UIUC



Learning Boolean Functions



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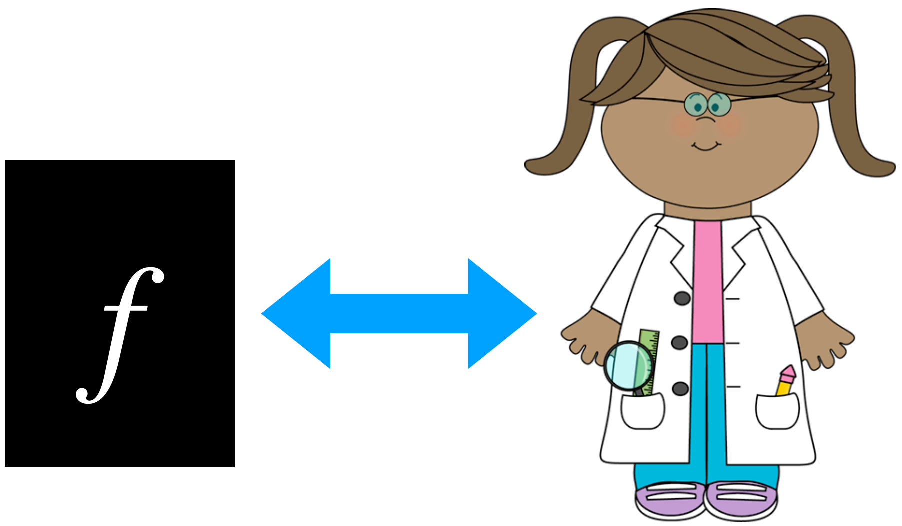

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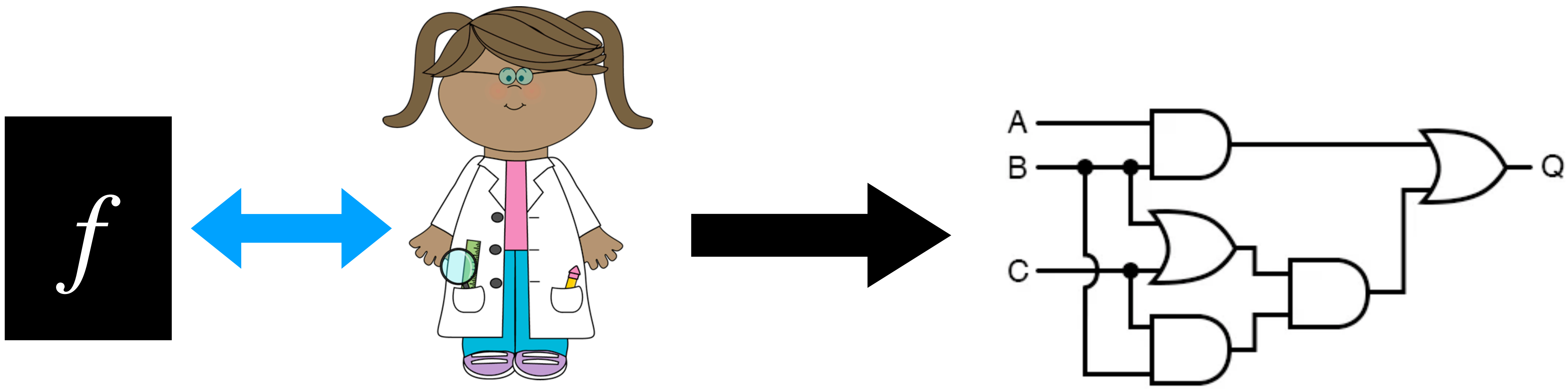

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2. Learner's goal is to output a function h that agrees with f on most inputs.

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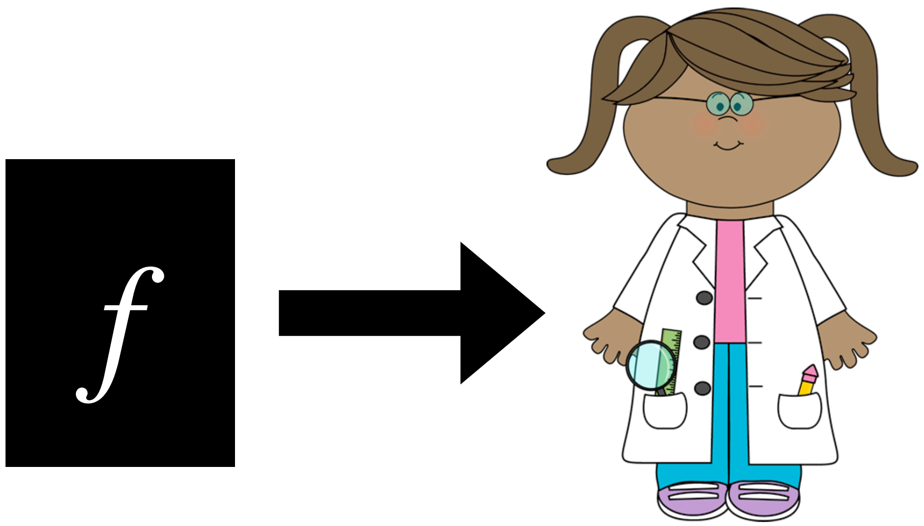
Improper: Learner can output any circuit, not necessarily in $\mathscr{H}$.

The Learning Model, Part II
Types of Function Access



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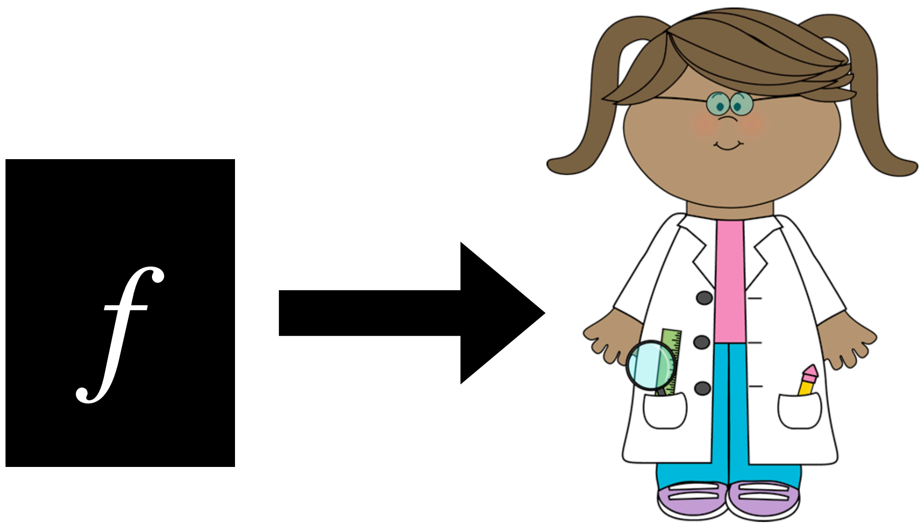

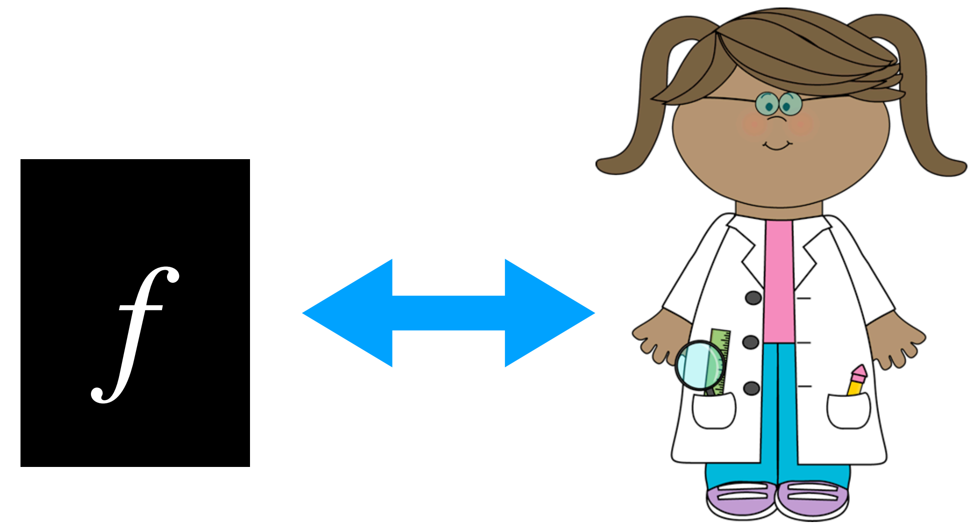

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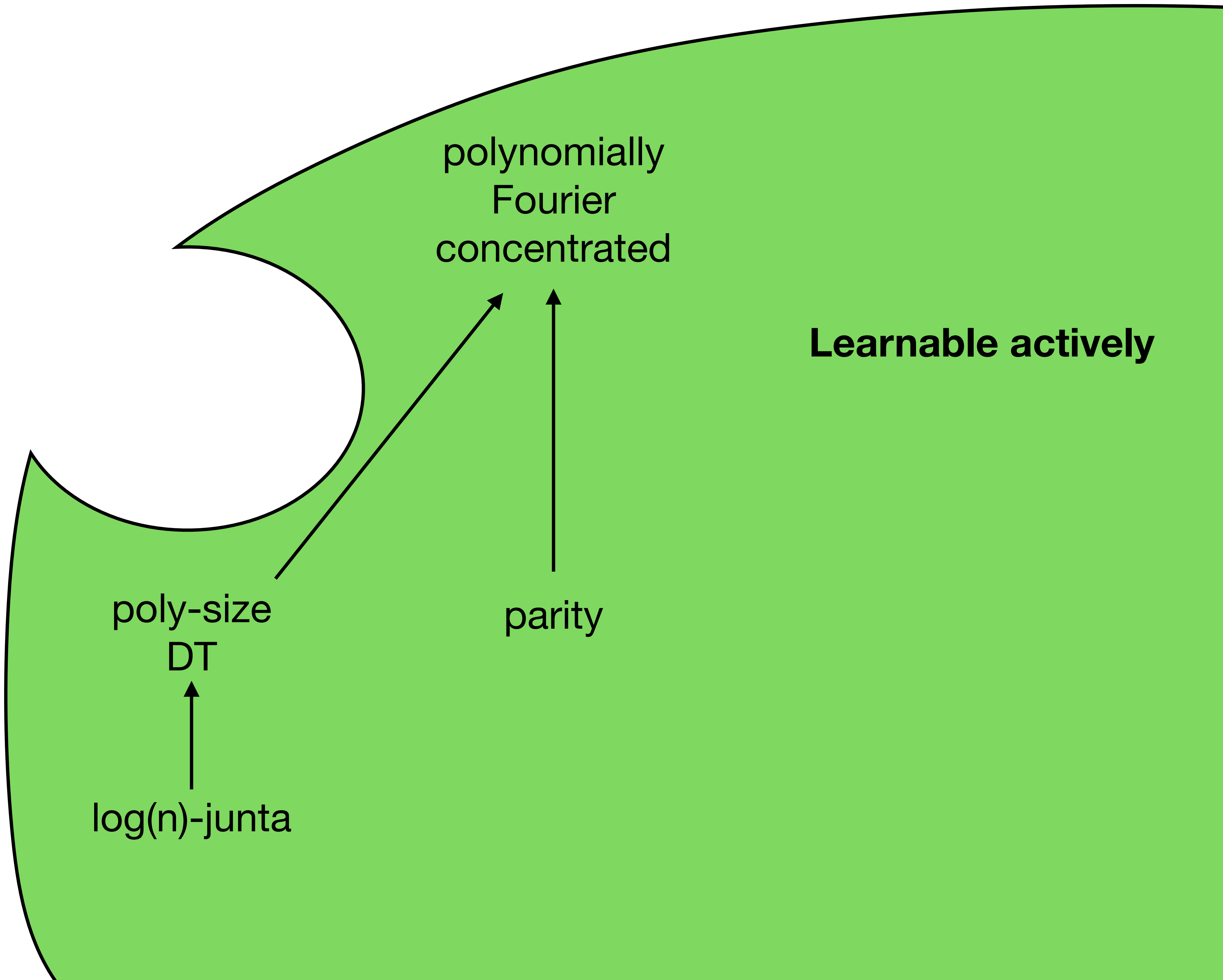
Active Learning:

Learner gets oracle access to f.

Passive vs. Active Learning



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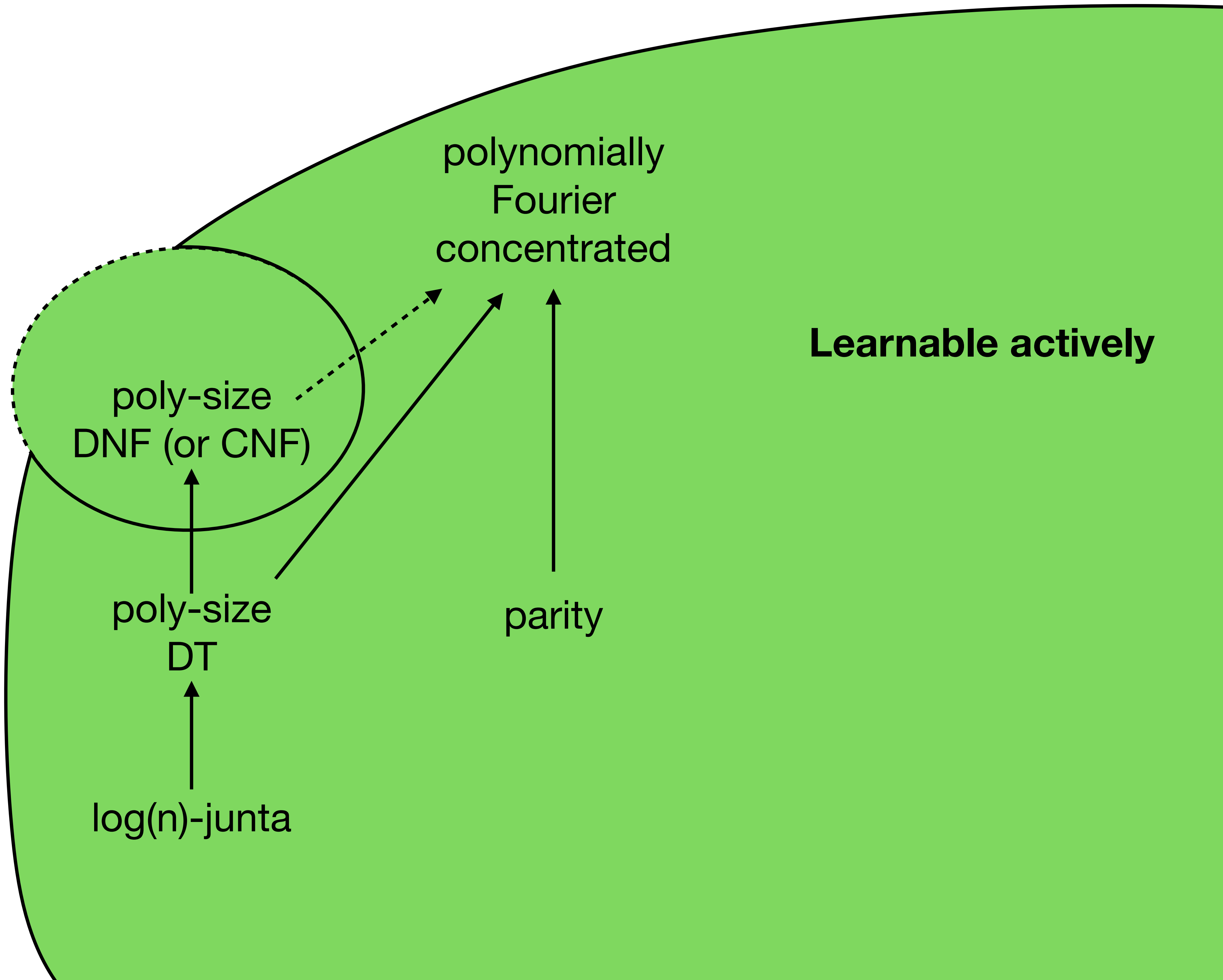

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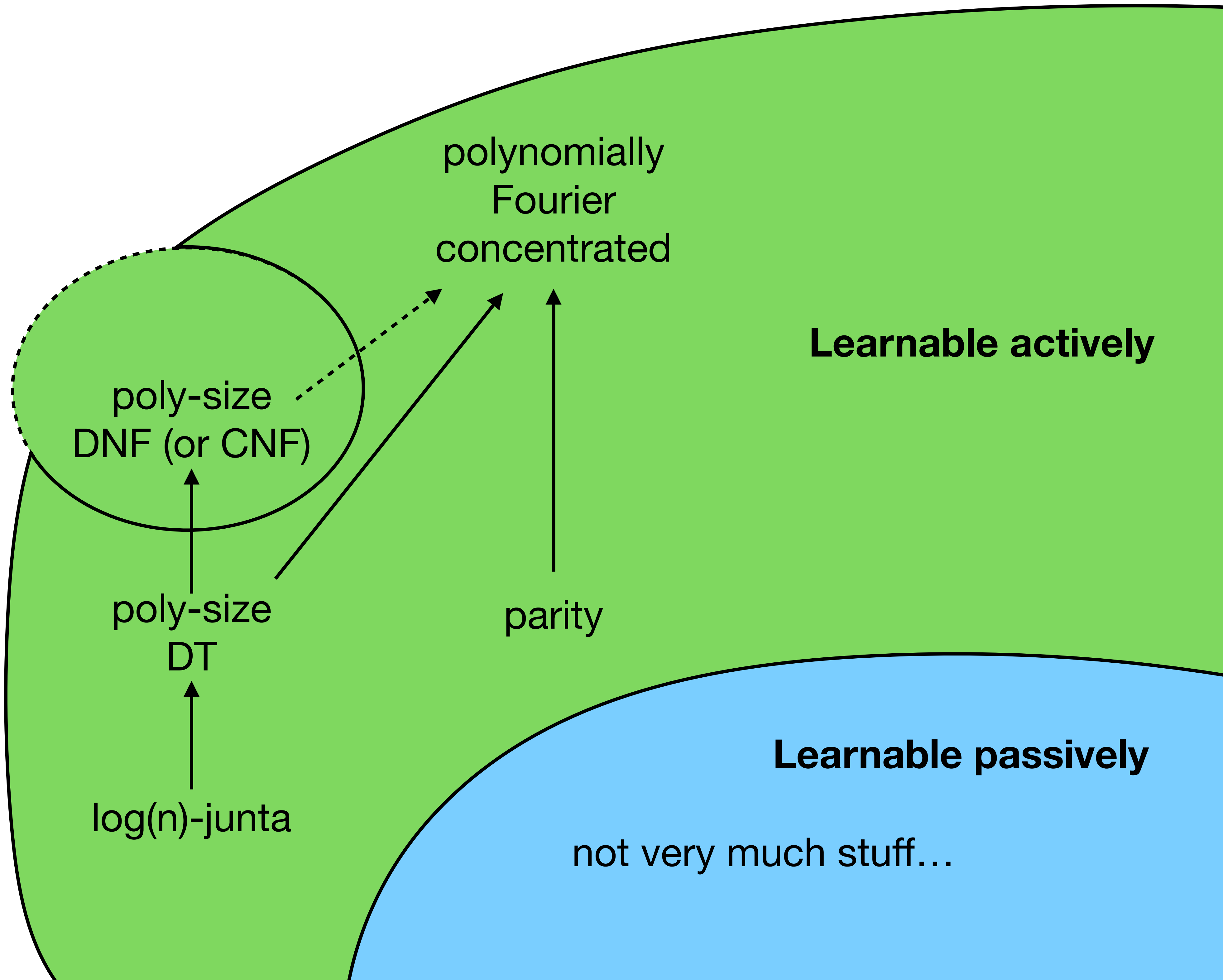

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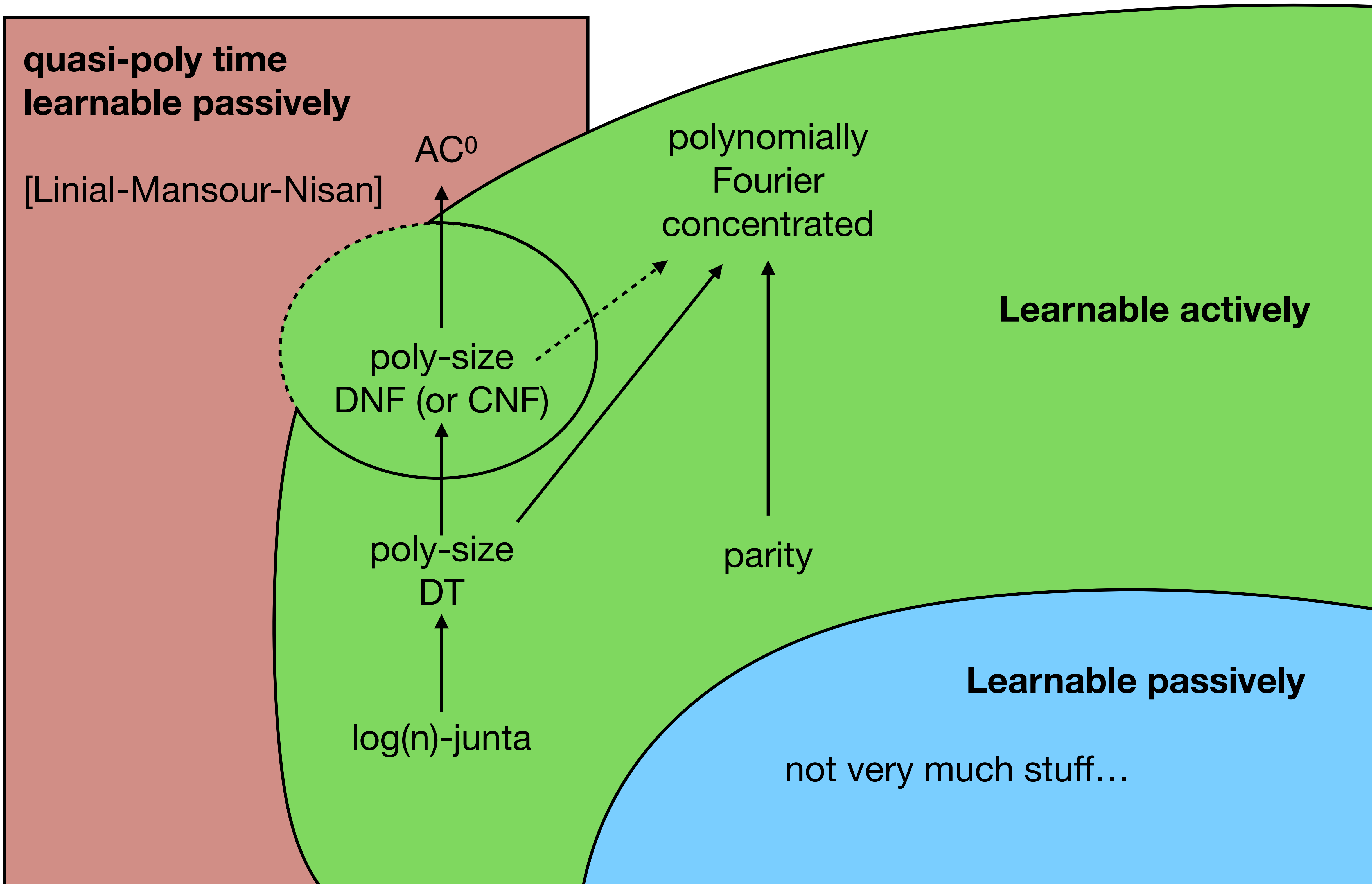

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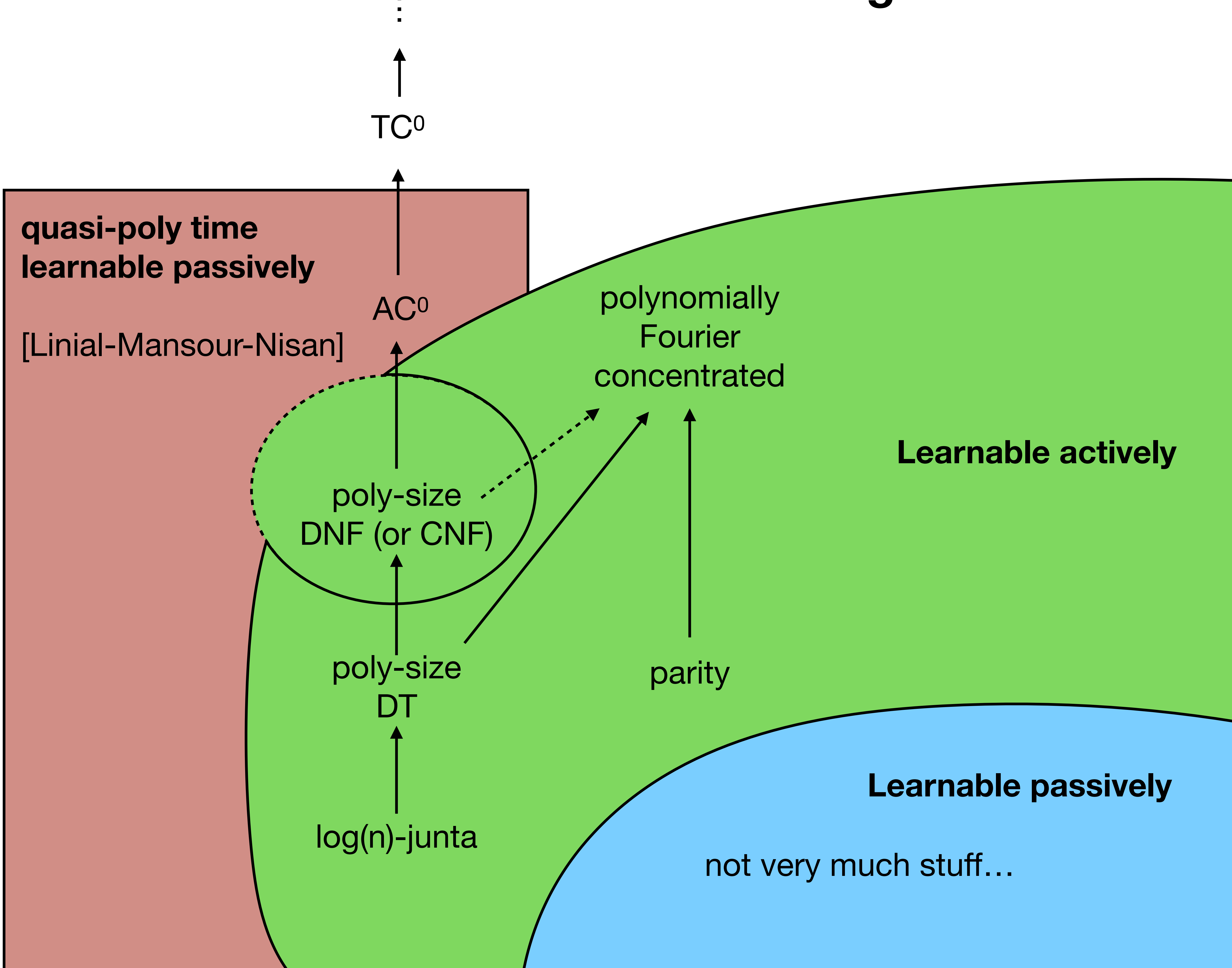

where’s the adversary?

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1. **Covert Learning**: [Canetti-Karchmer '21, IKOS '19]
   curious eavesdropper tries to piggyback on the queries of an active learner.

2. **Verifiable Learning:** [Goldwasser-Rothblum-Shafer-Yehudayoff '20]
   untrusted prover claims that a hypothesis h approximates f near-optimally (compared to some class of functions).

Covert Learning

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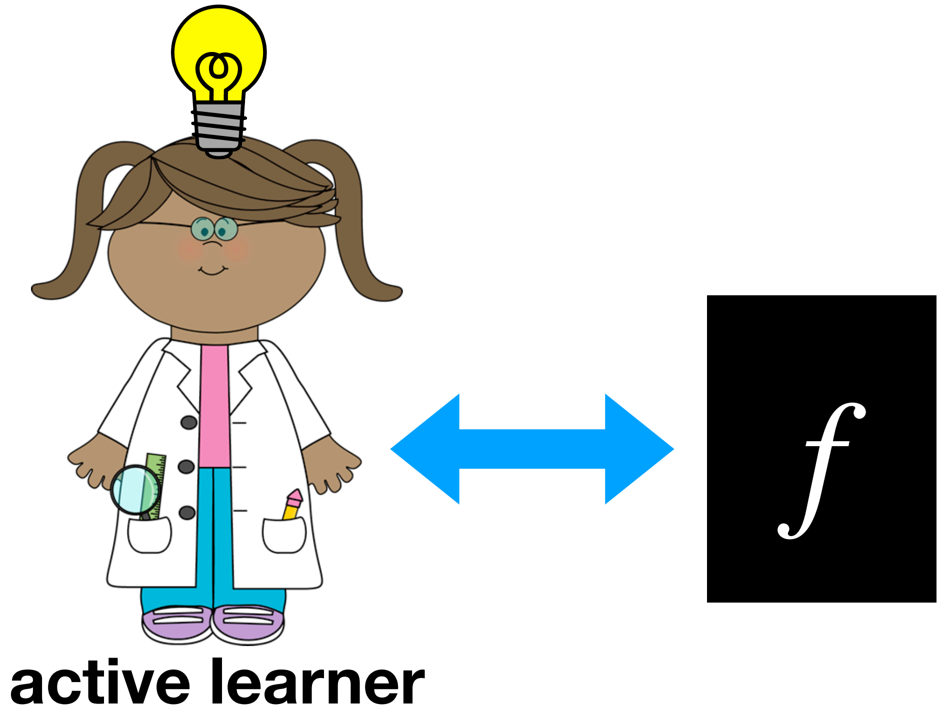

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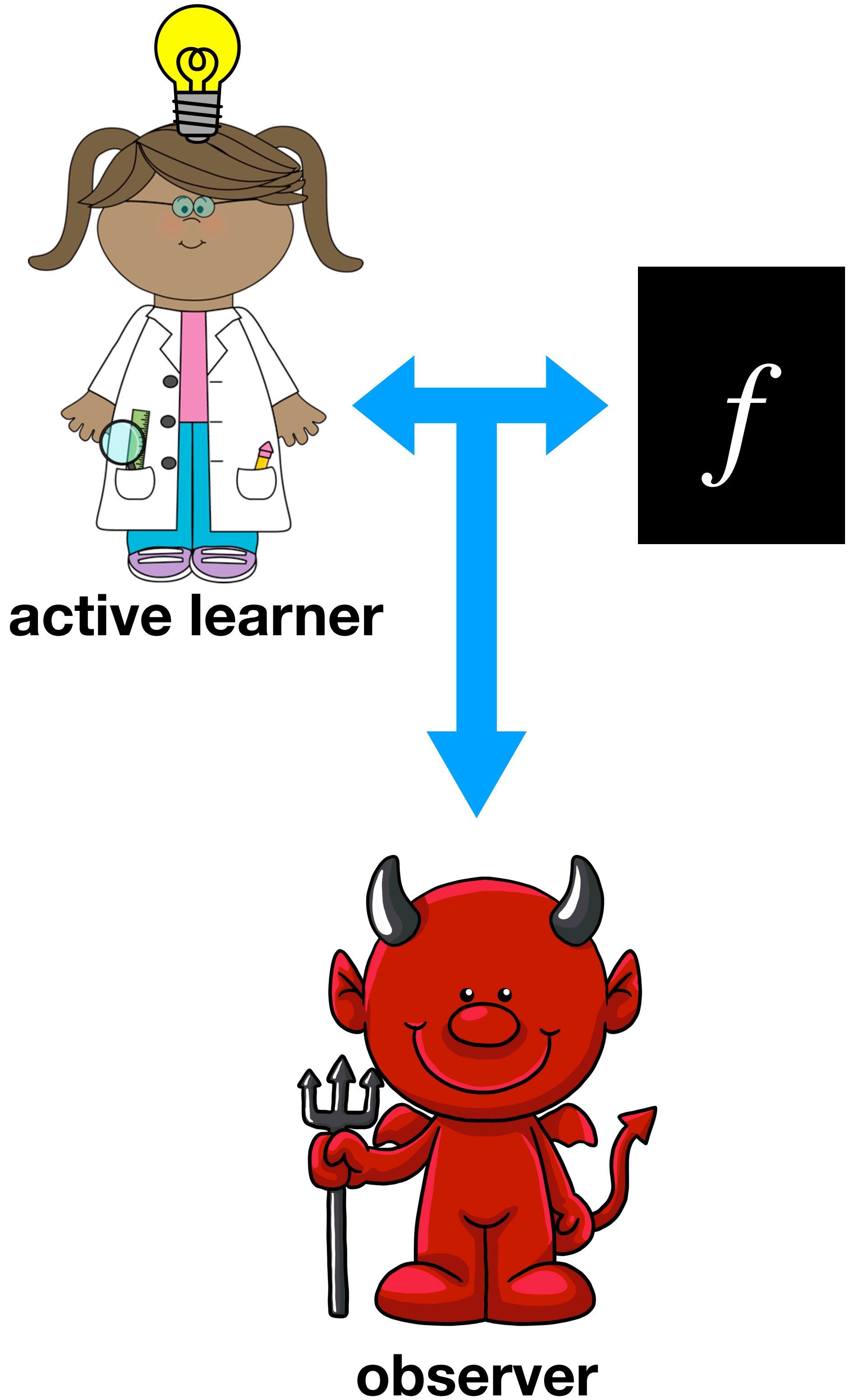

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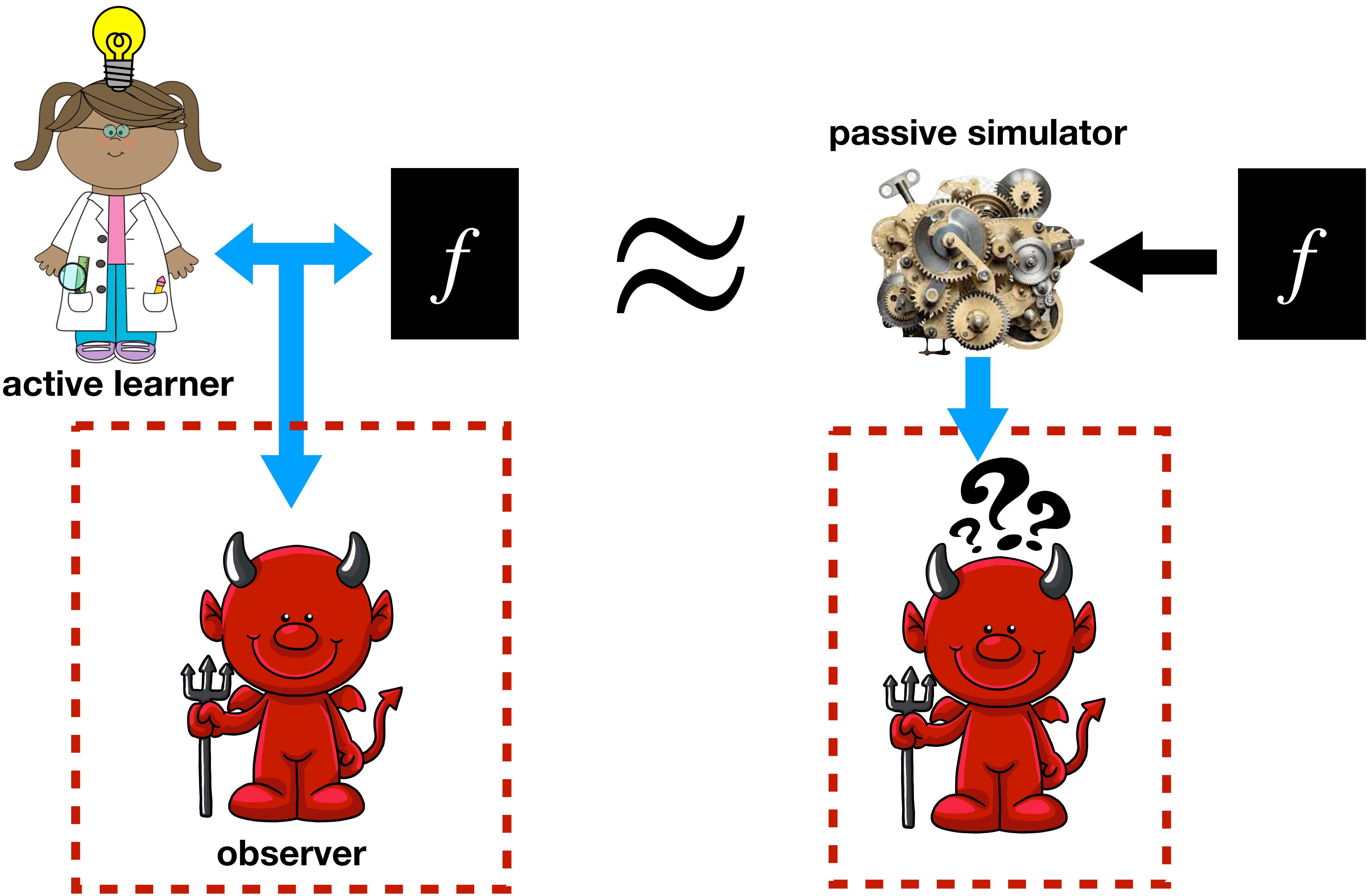

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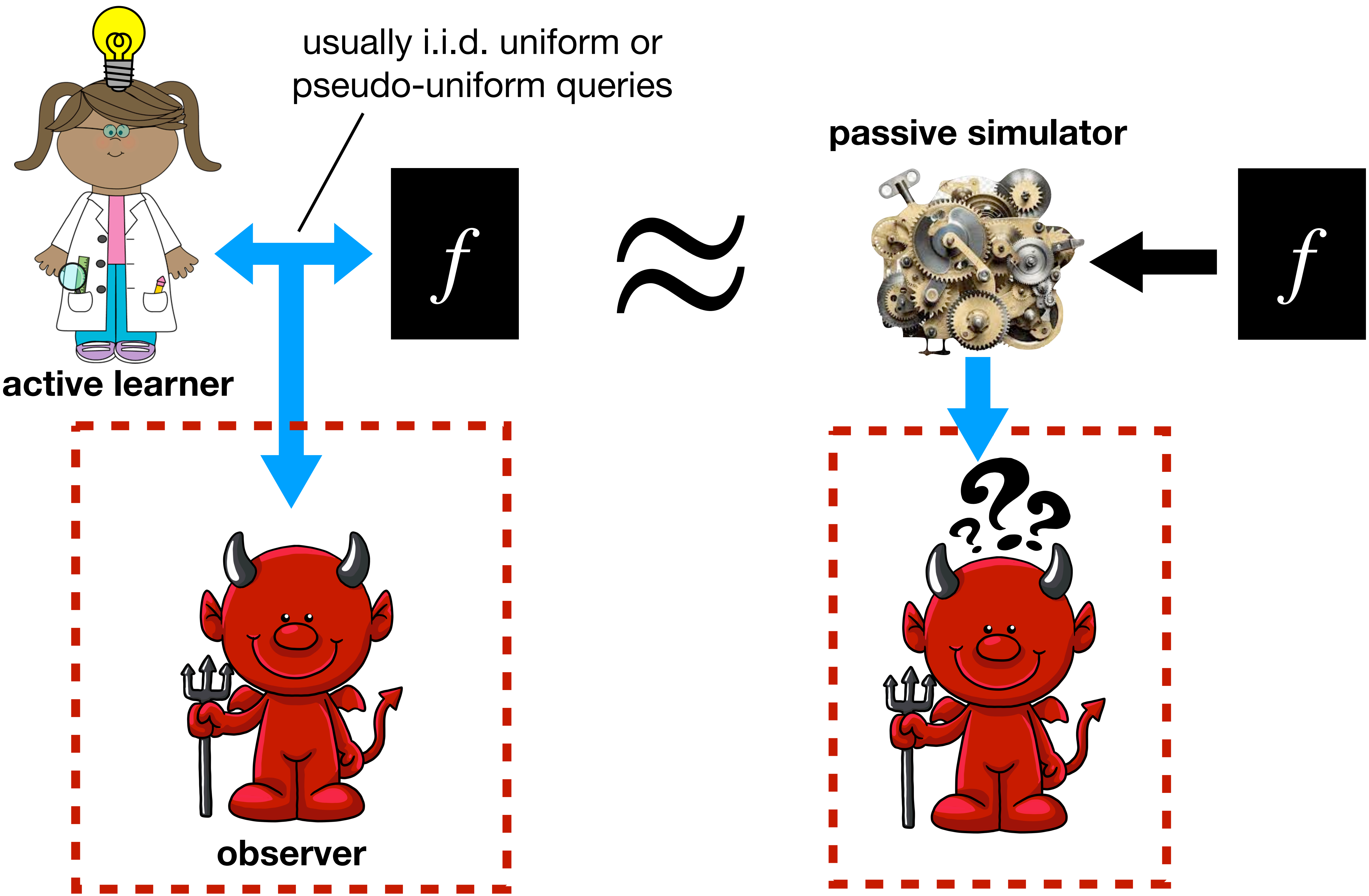

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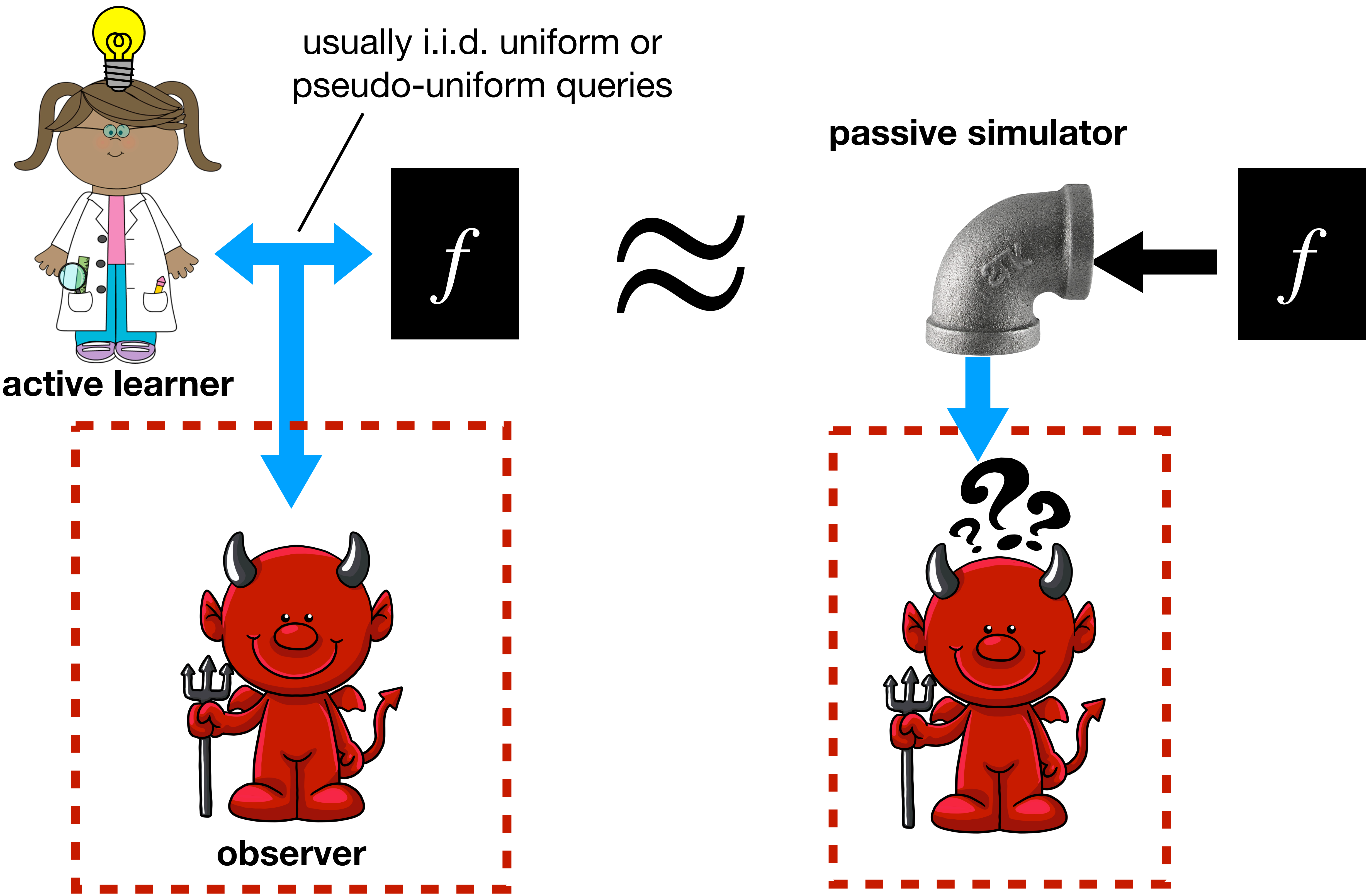

(1-of-2) Locally Covert Learning

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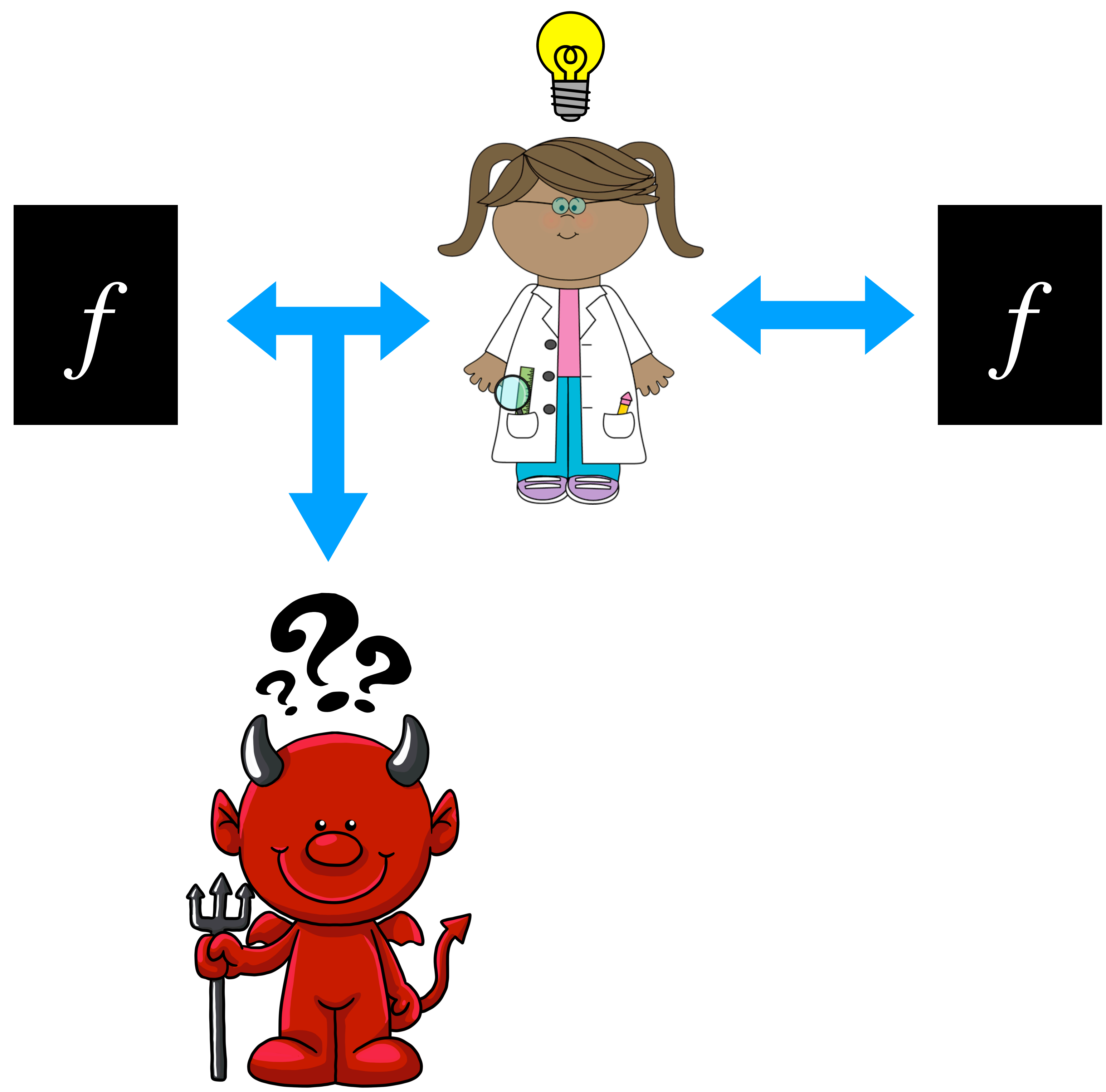

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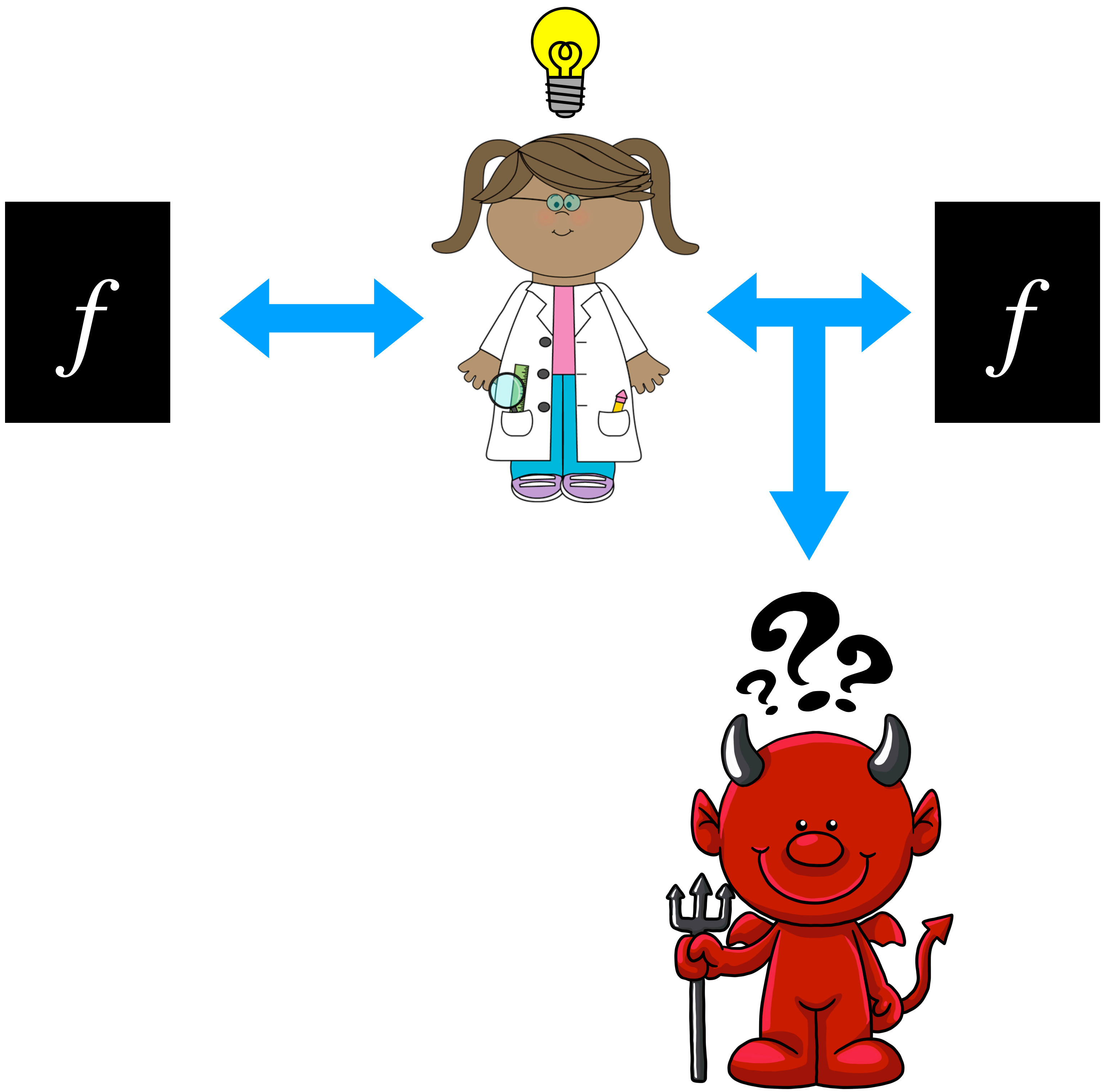

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Solution: use covert learning $\implies$ their data has no resale value

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➡ If learning algorithm is also "robust" then a few incorrect query answers can't ruin the output.

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Can’t really work against a covert learner 😢

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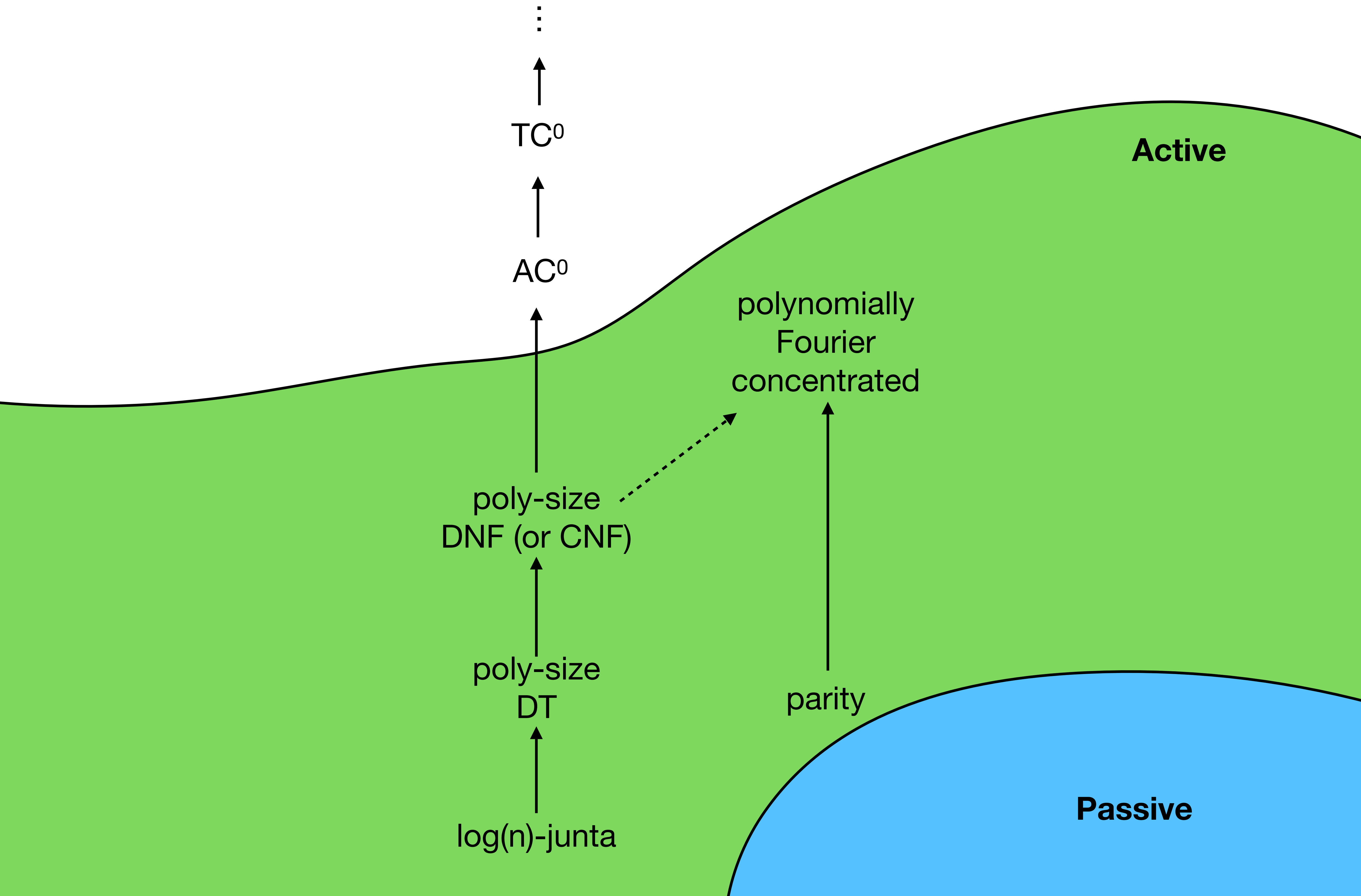

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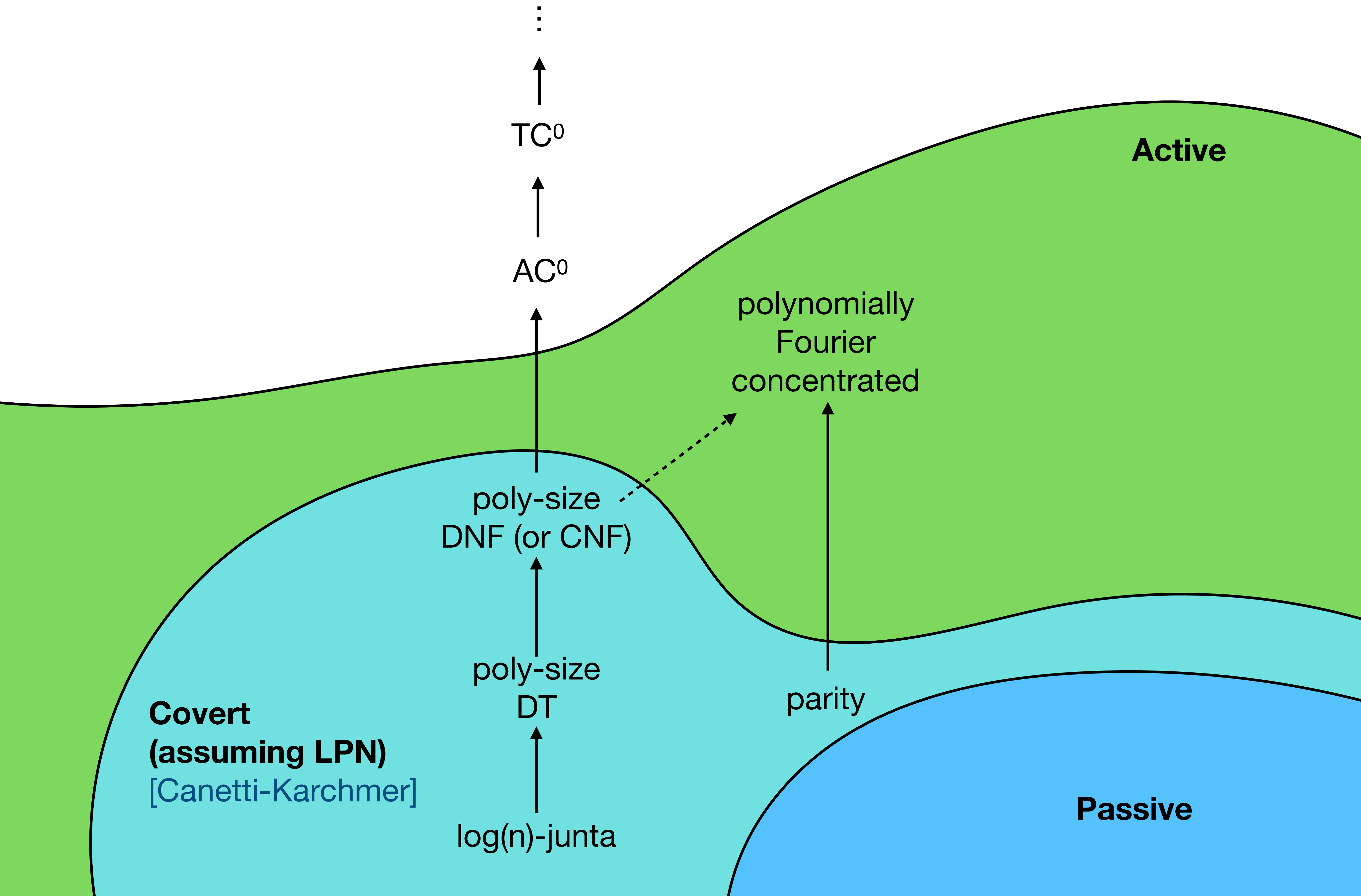

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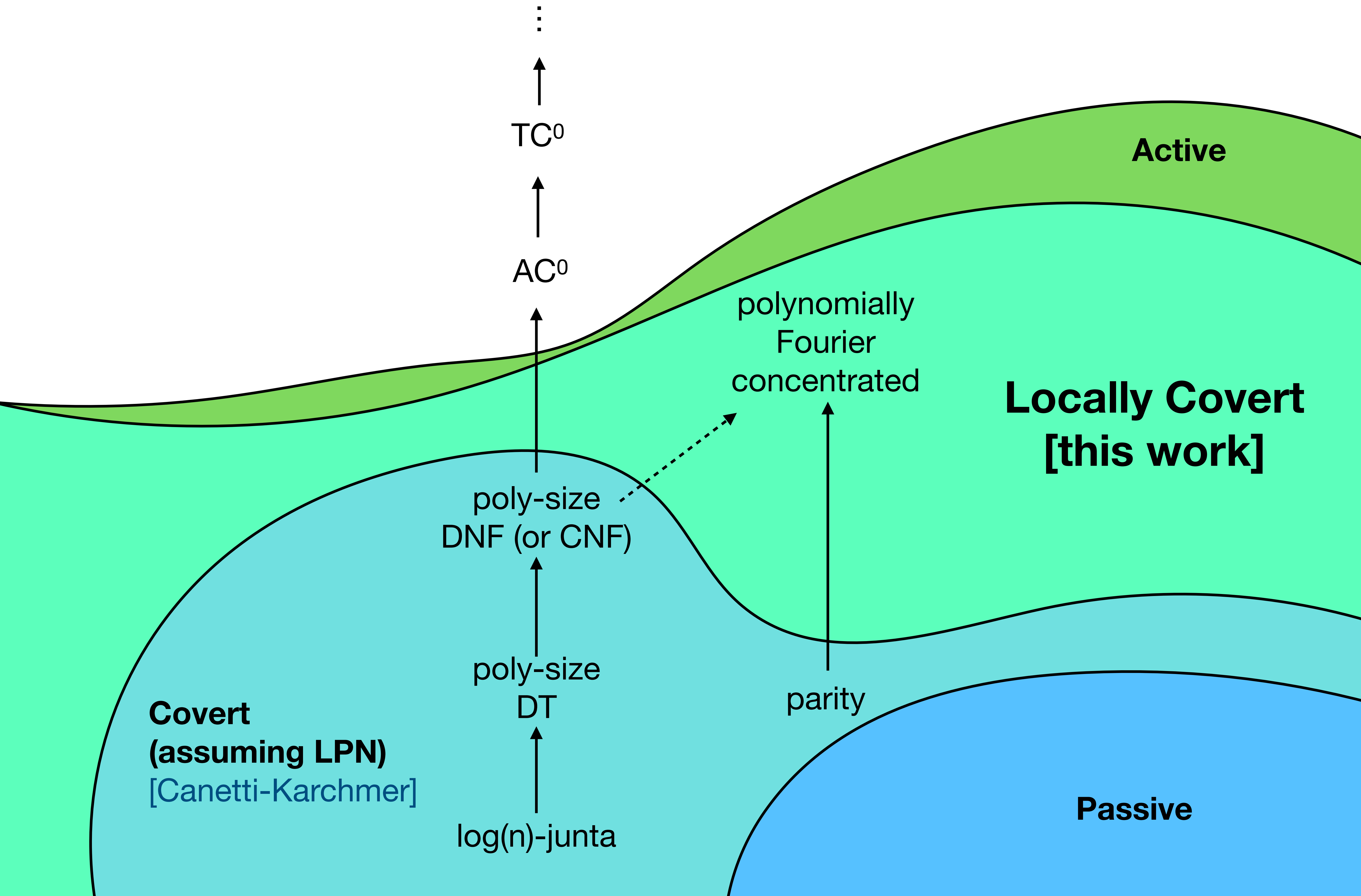

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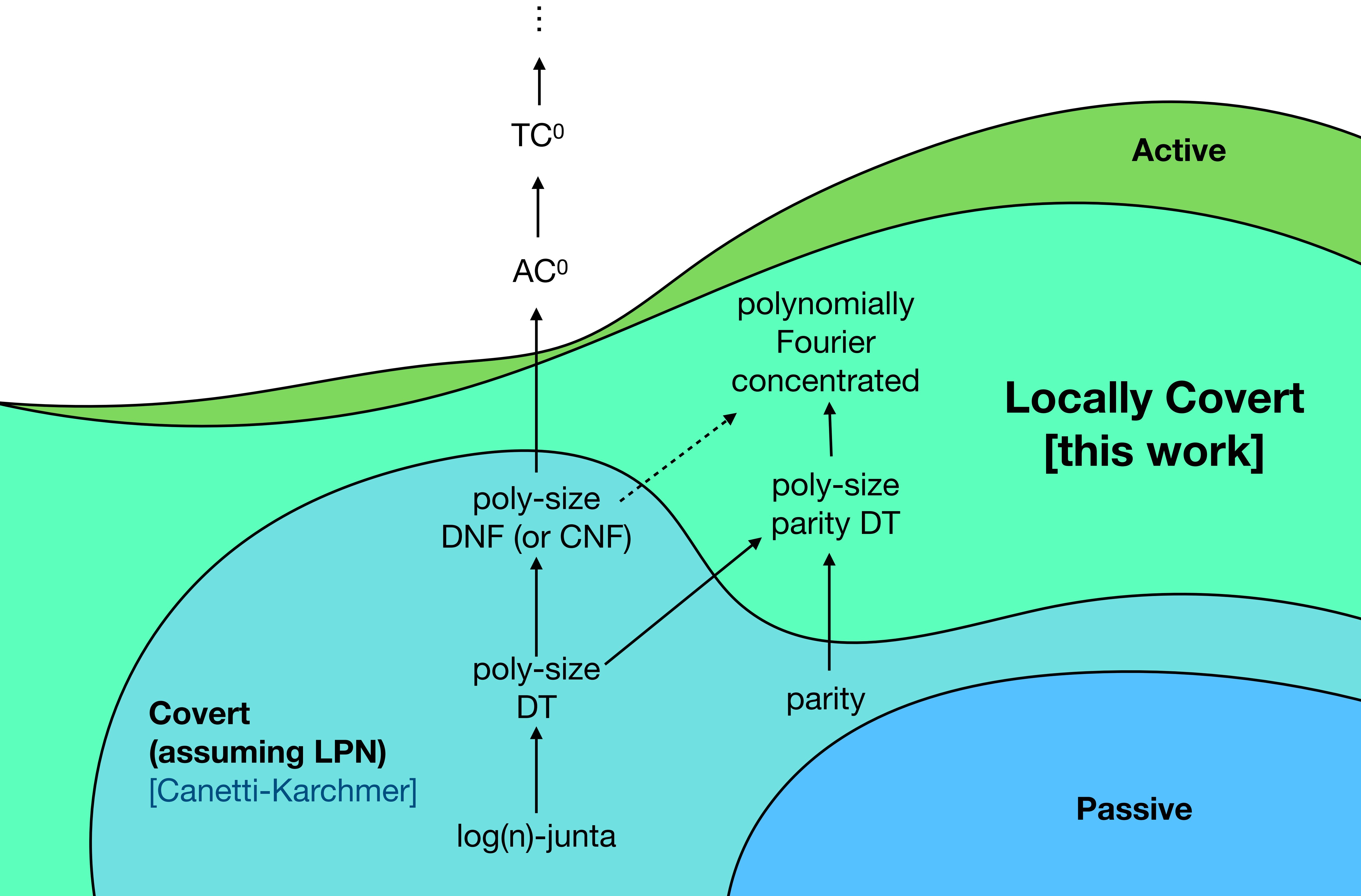

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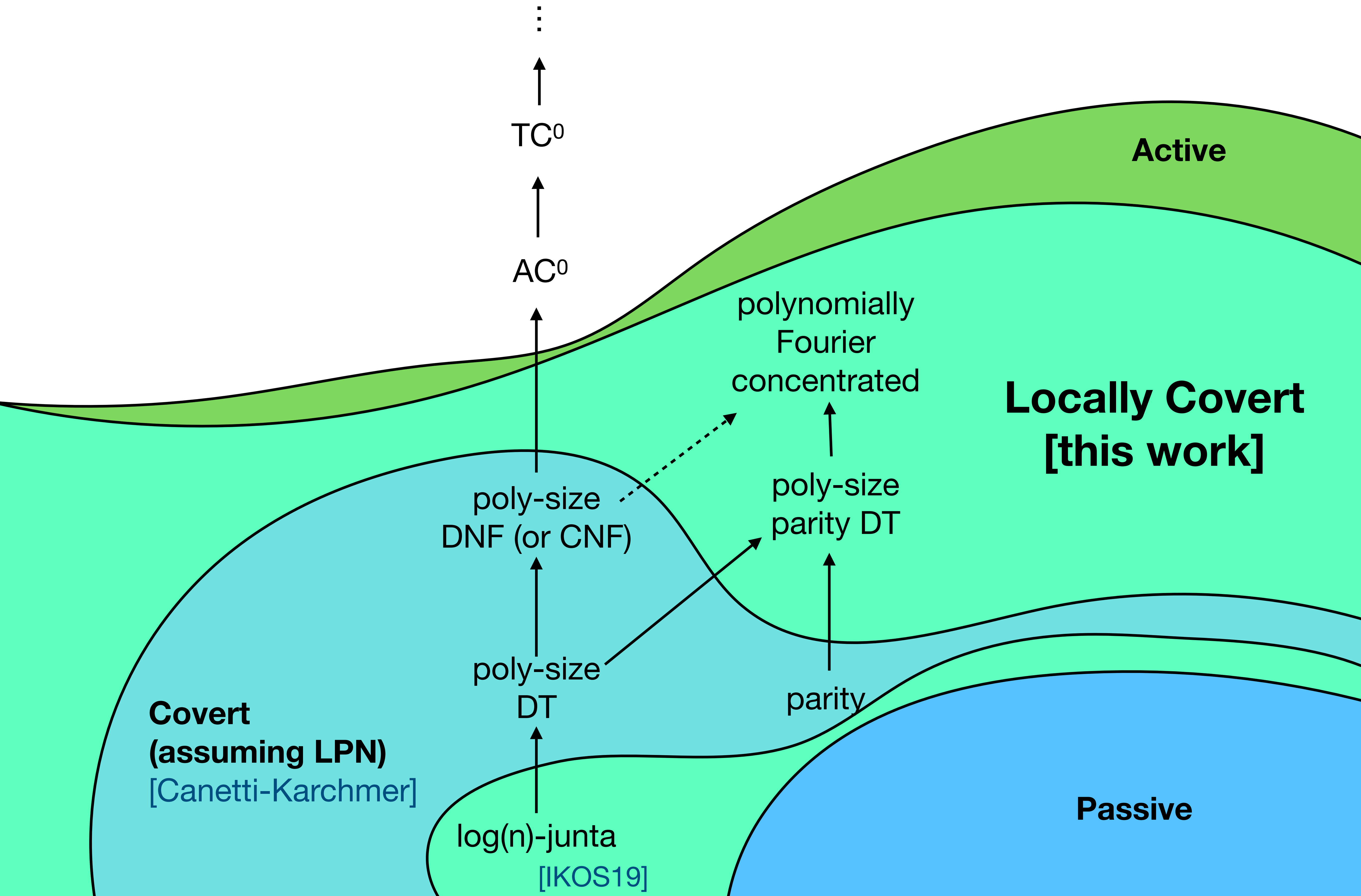
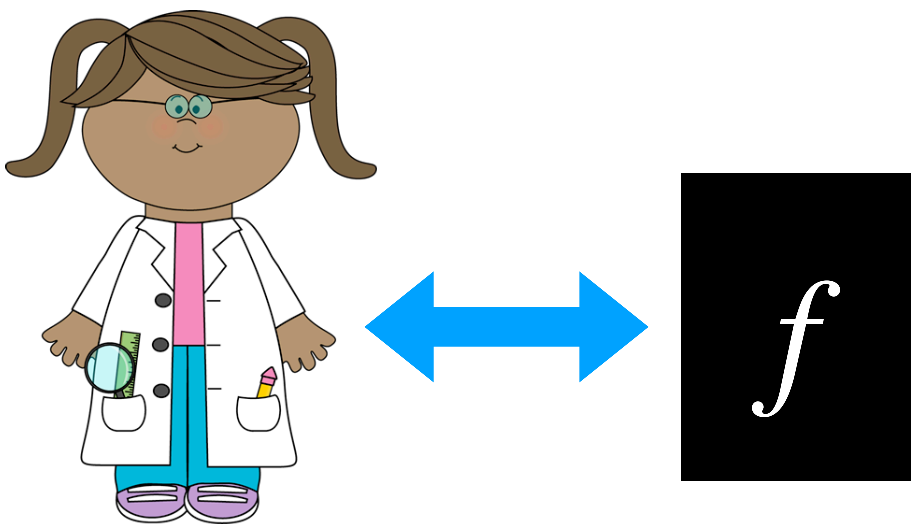


The Goldreich-Levin Theorem



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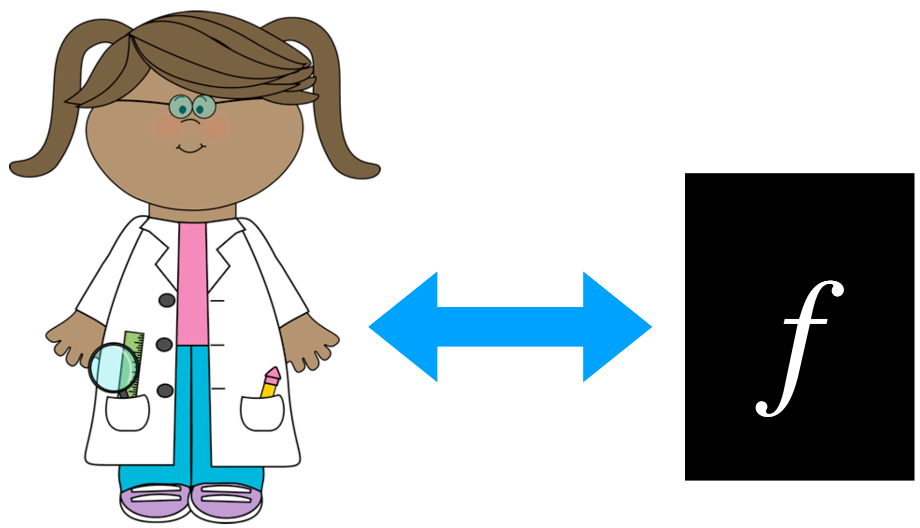

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Let g be a OWF. Then $\langle \mathbf{x}, \mathbf{r} \rangle \pmod 2$ is *hard-core* for $(g(\mathbf{x}), \mathbf{r})$.

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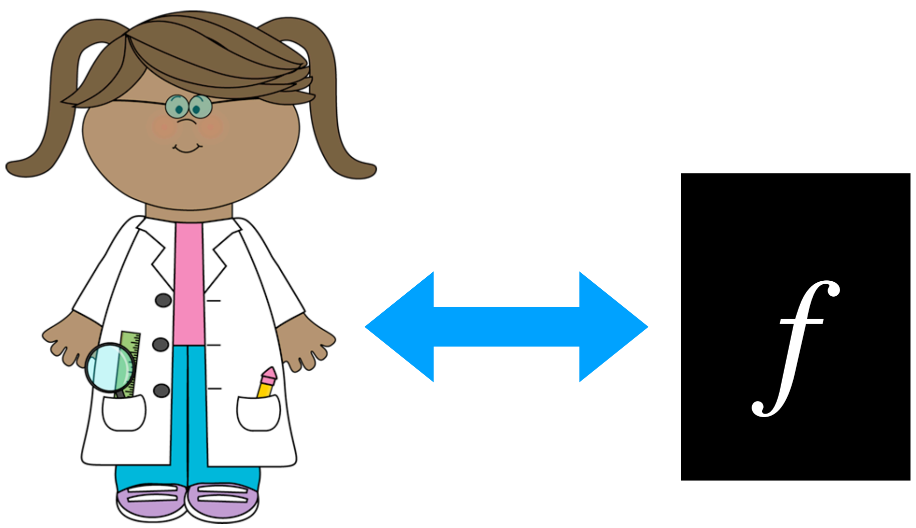

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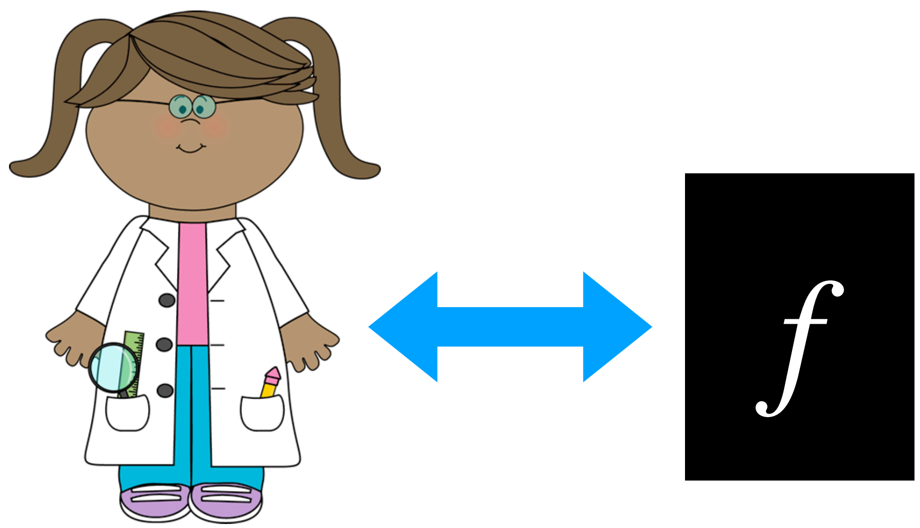

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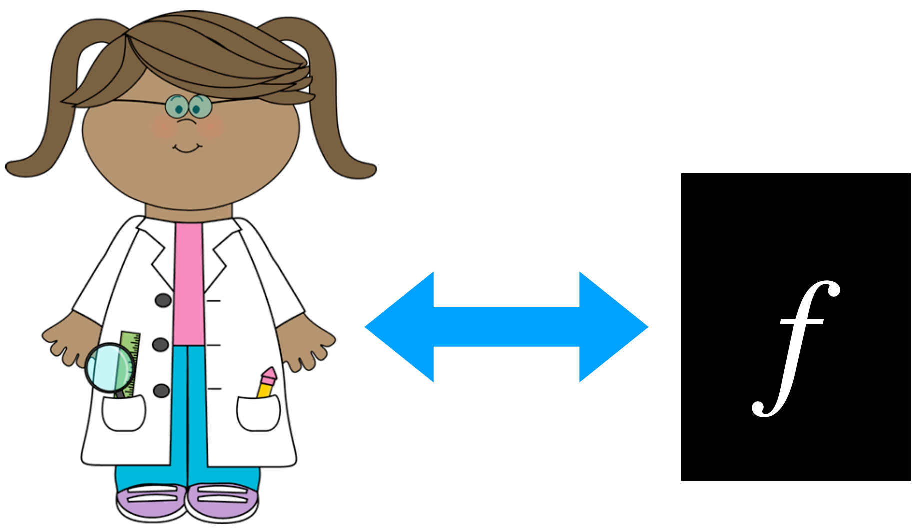

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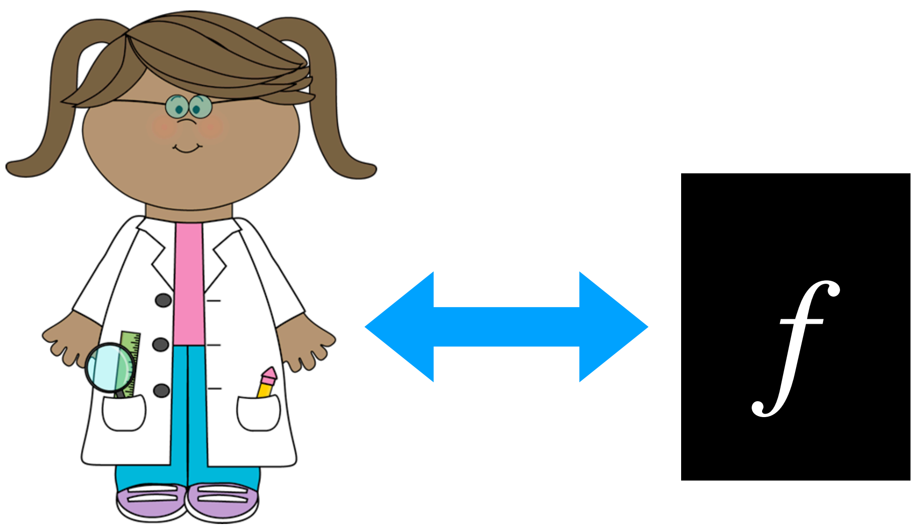

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Small modification gives $(k-1)$-out-of-k covertness.

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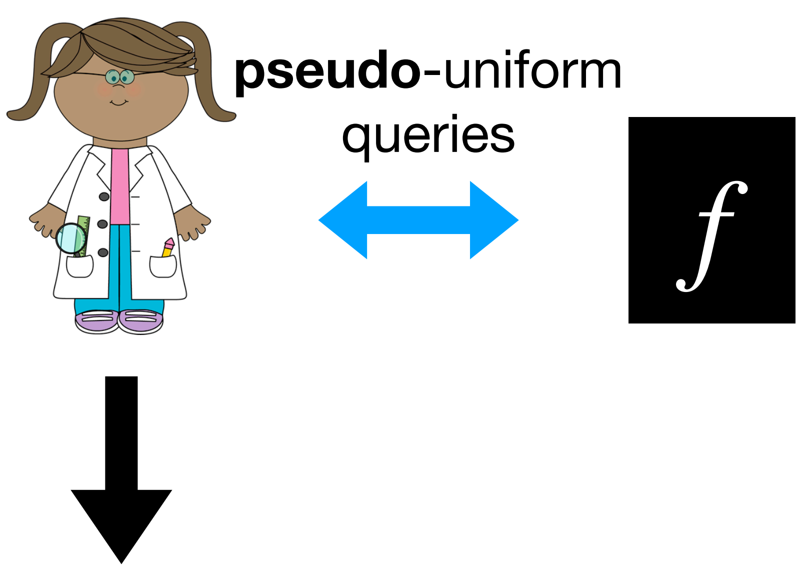

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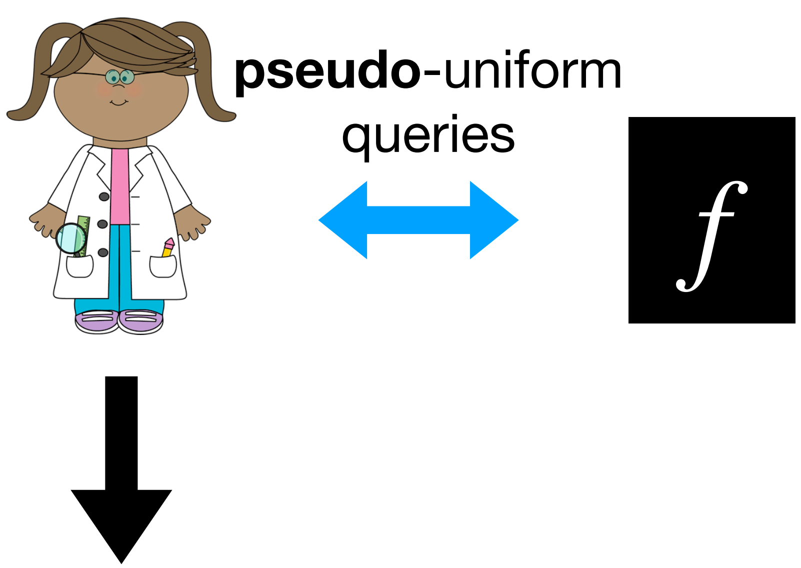

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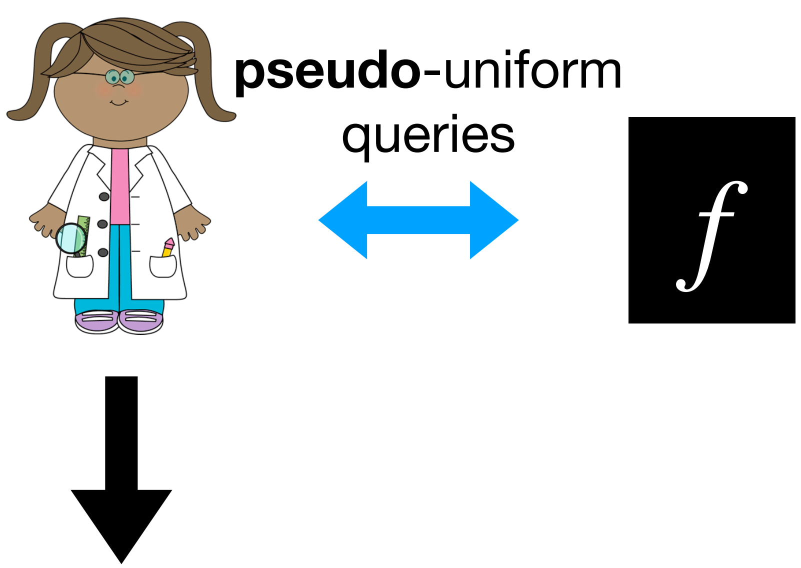

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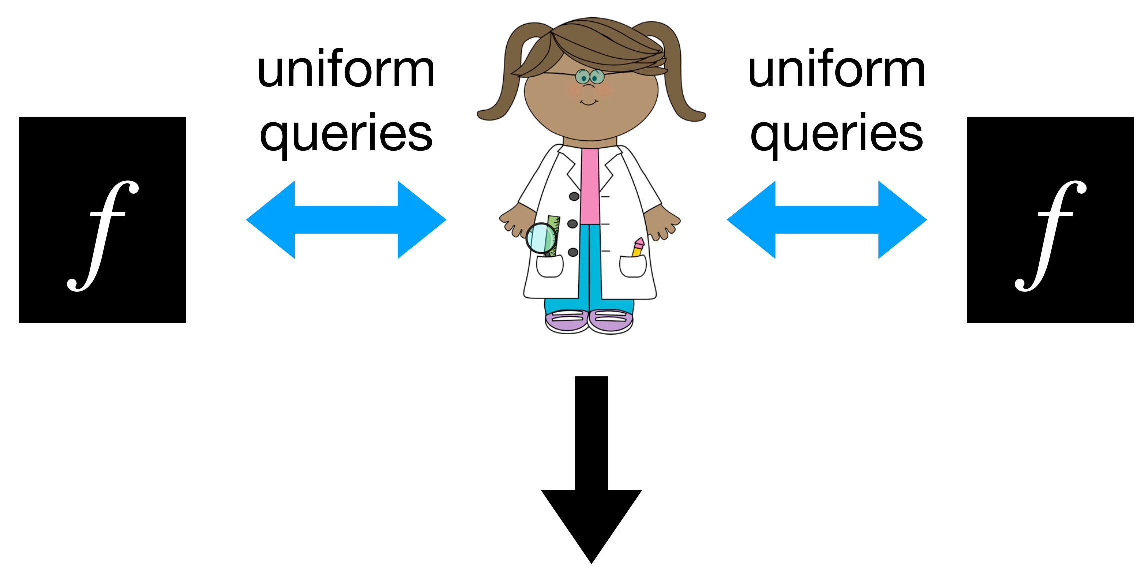

all γ s.t. $|\hat{f}(\gamma)| \geq \epsilon$
(except with δ probability)

Fourier Analysis Essentials

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will prove this later

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$(k-1)$-wise uniform

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Goldreich-Levin with k-weights
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running time is exponential in k

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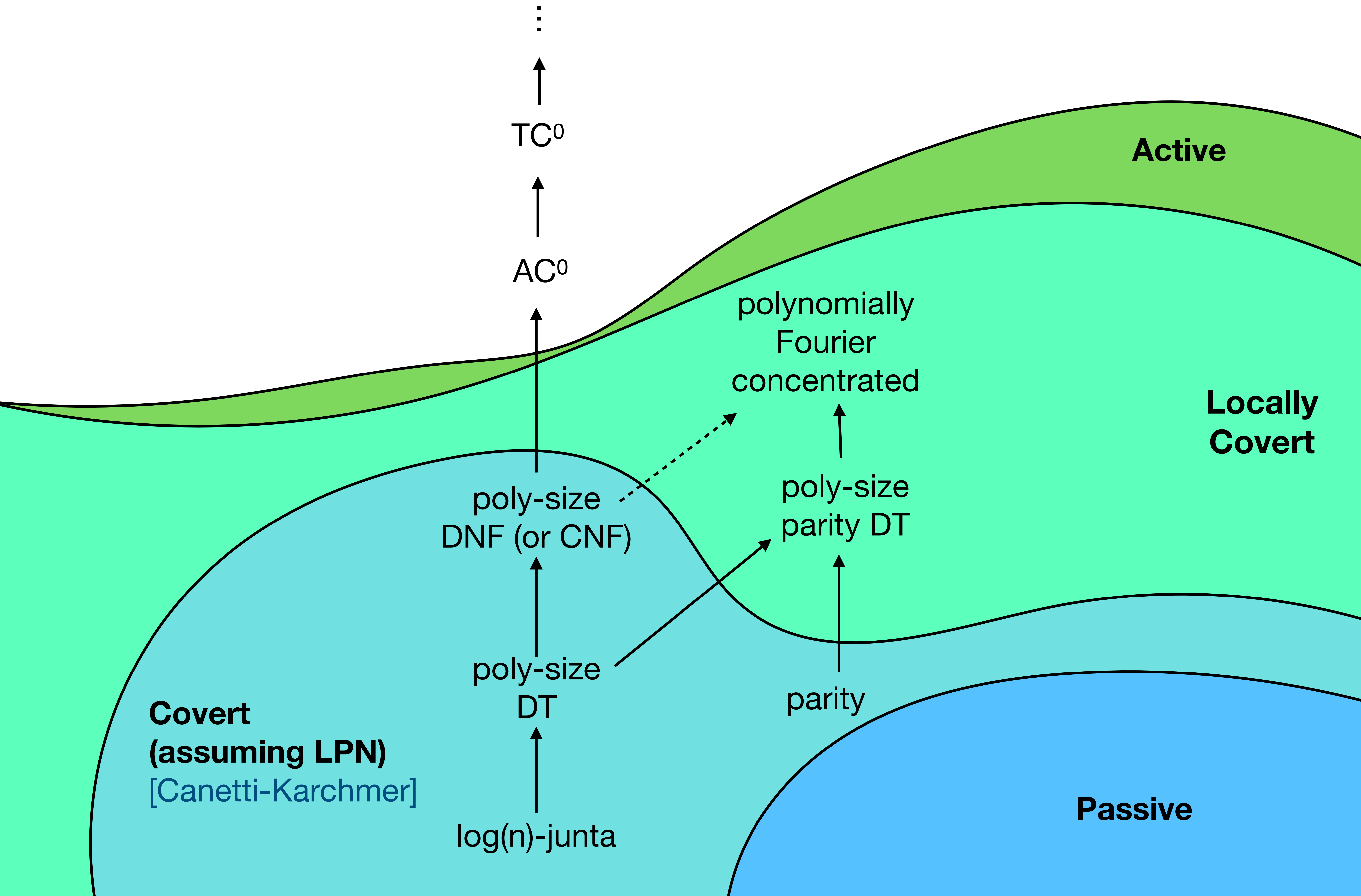
Future Work?


⋮
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AC⁰
polynomially
Fourier
concentrated
Active
Locally
Covert
poly-size
DNF (or CNF)
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(assuming LPN)
[Canetti-Karchmer]
log(n)-junta
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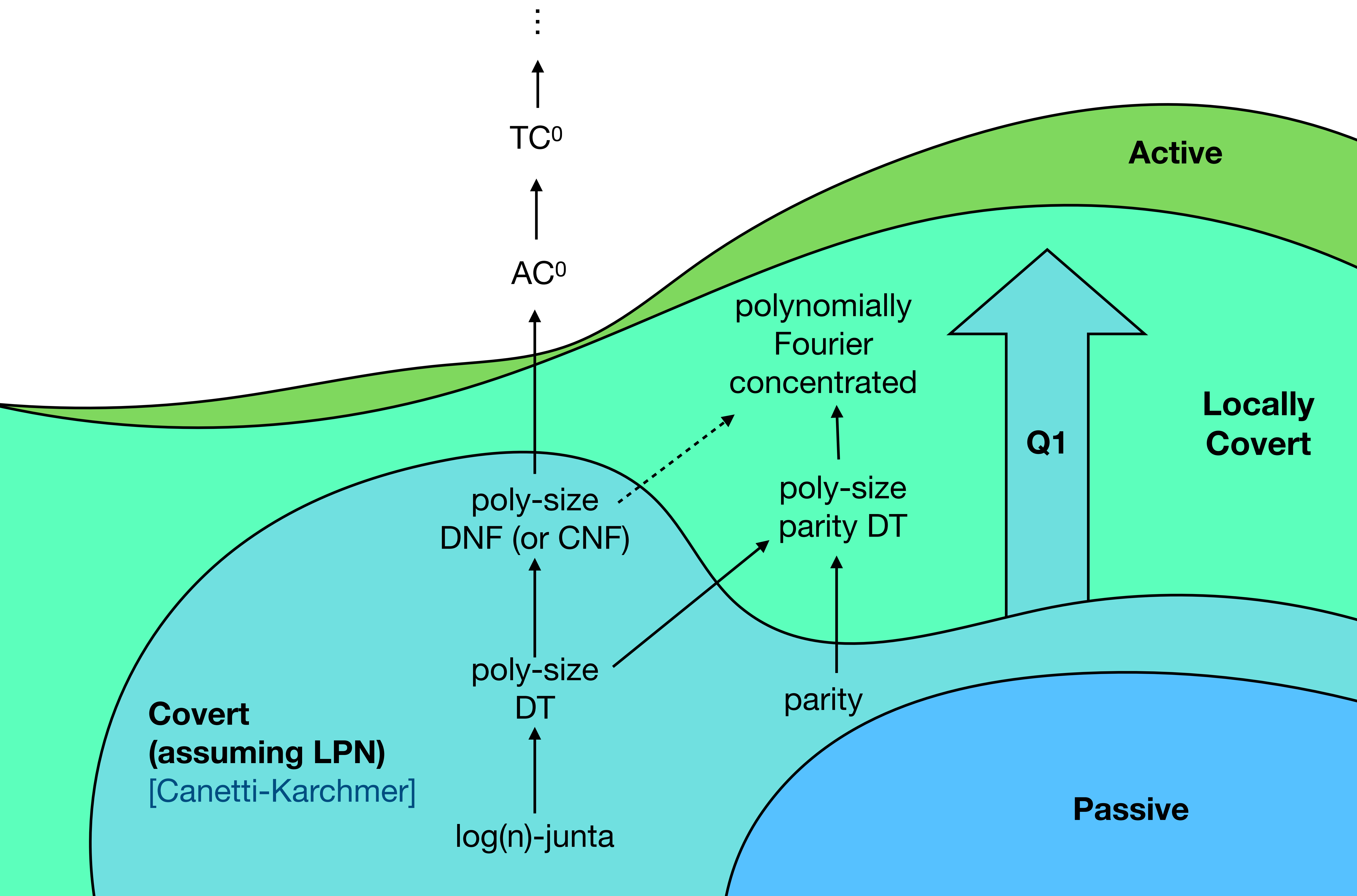
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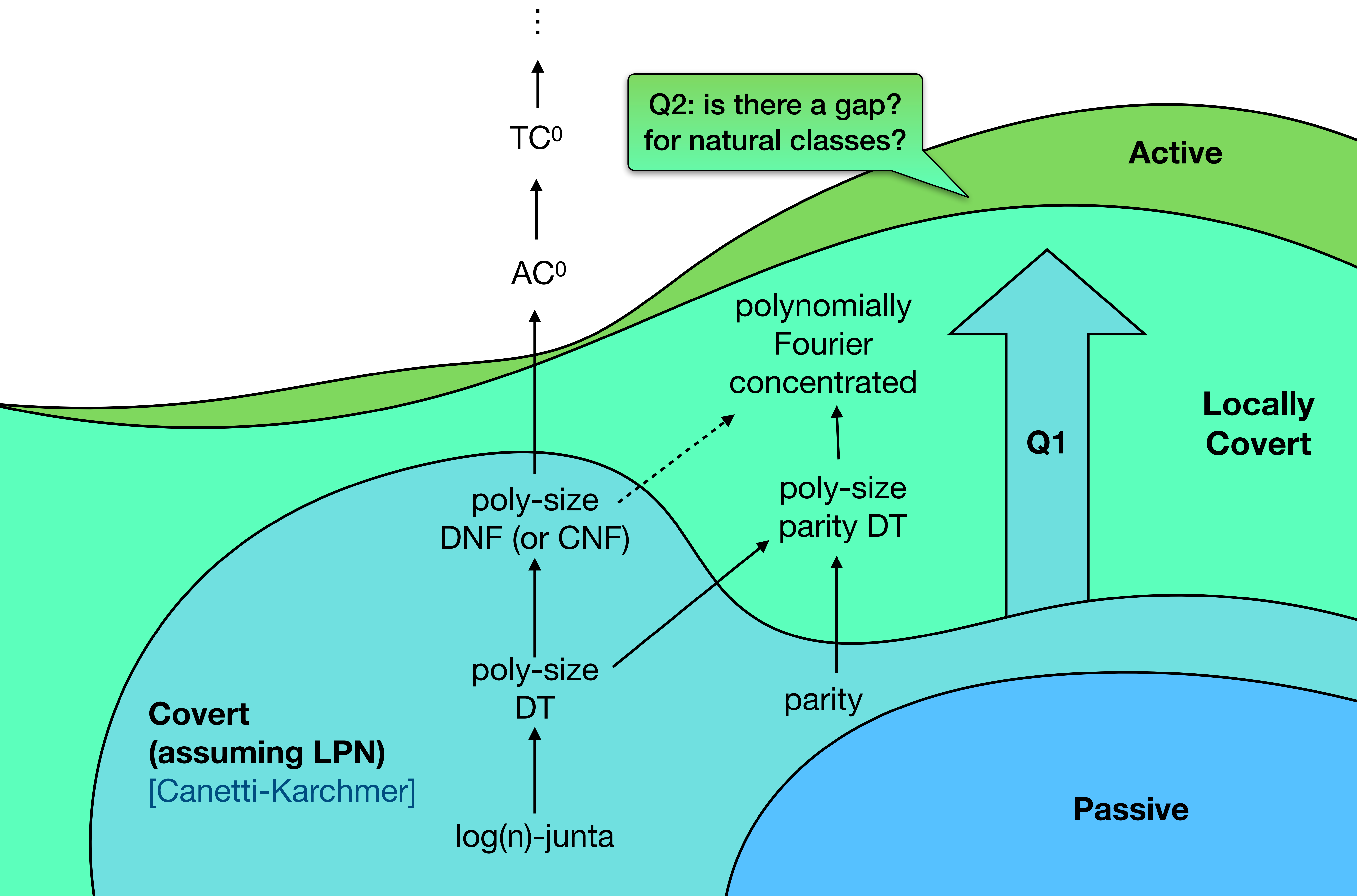

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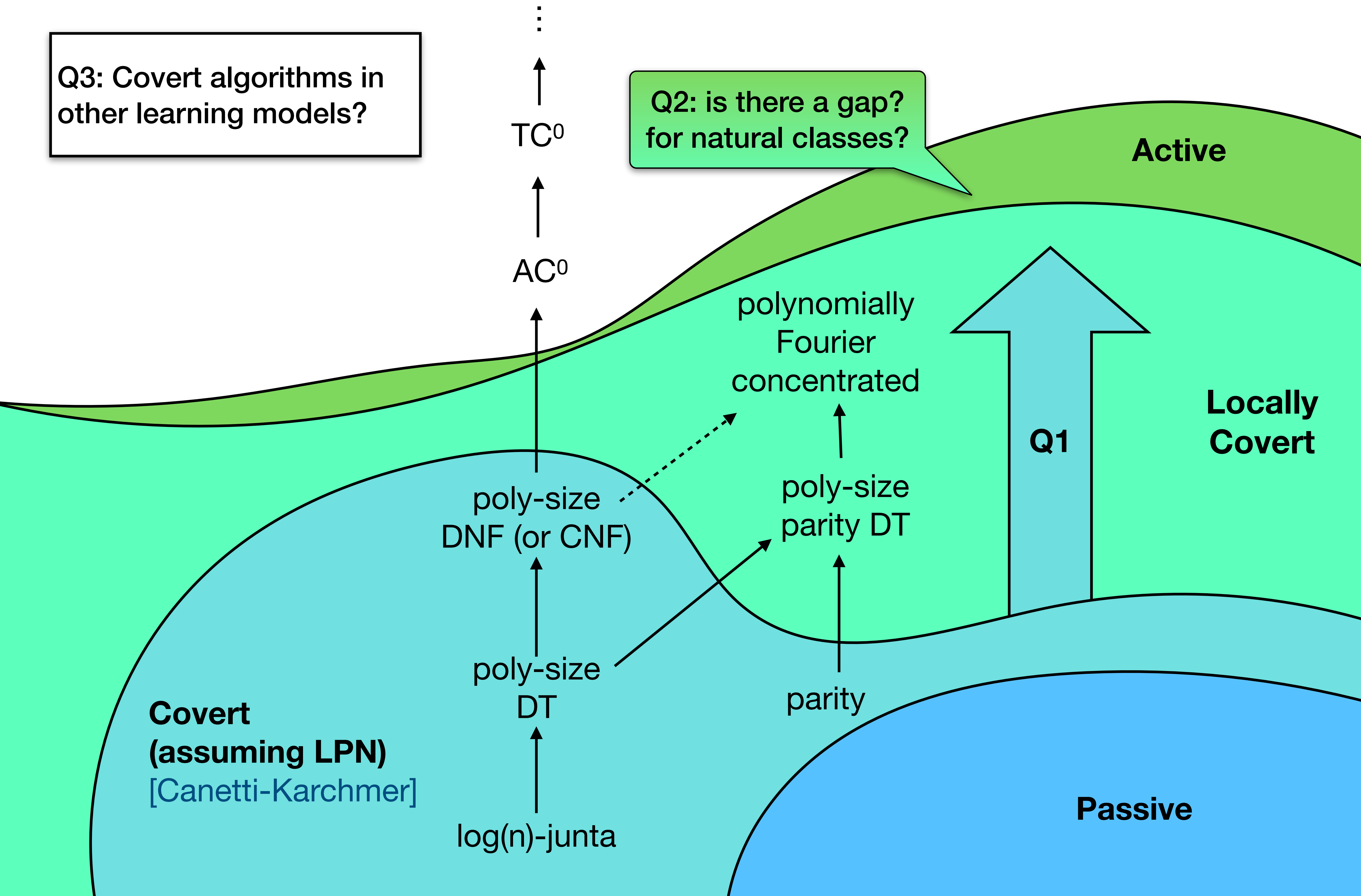
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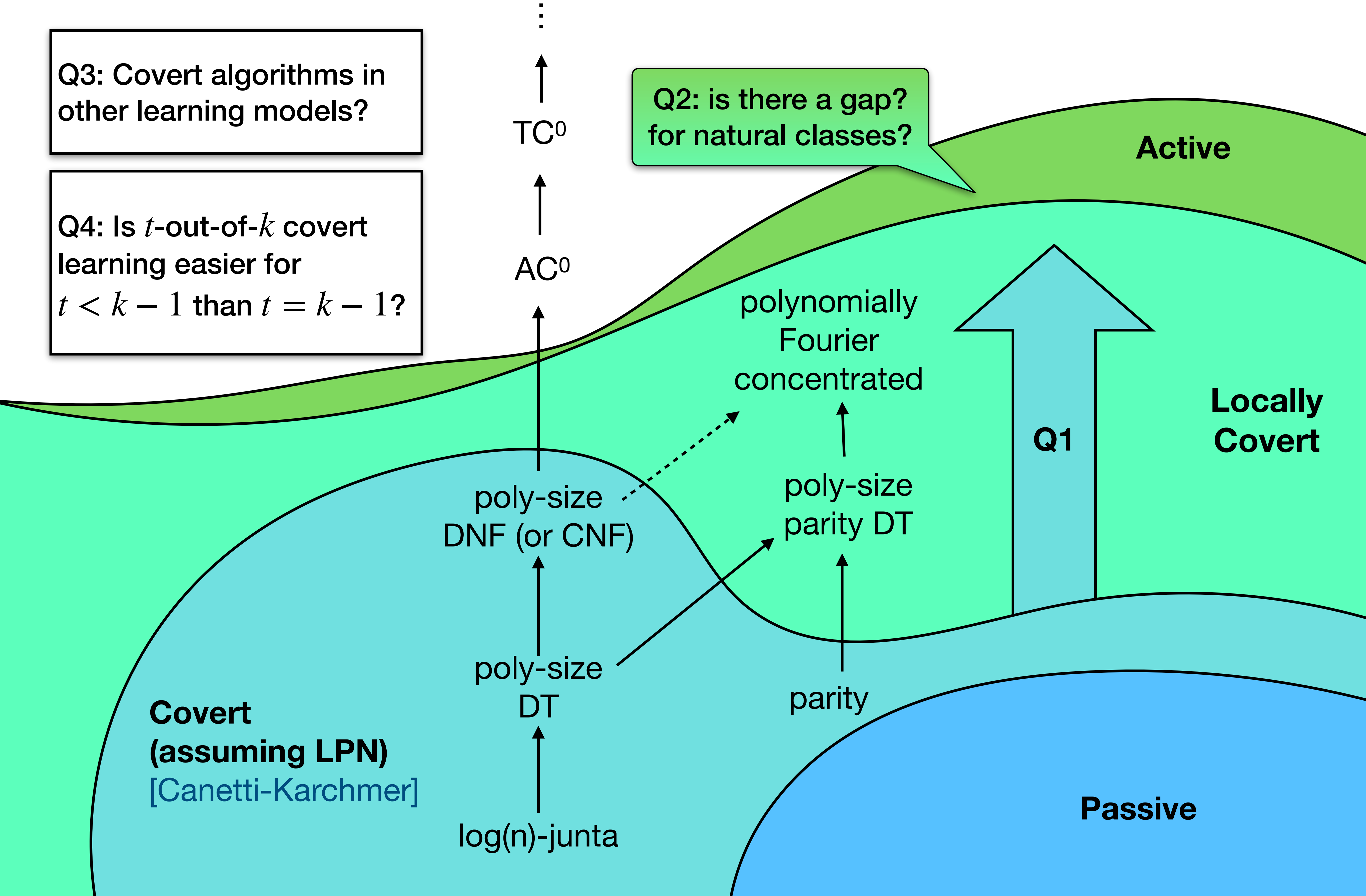
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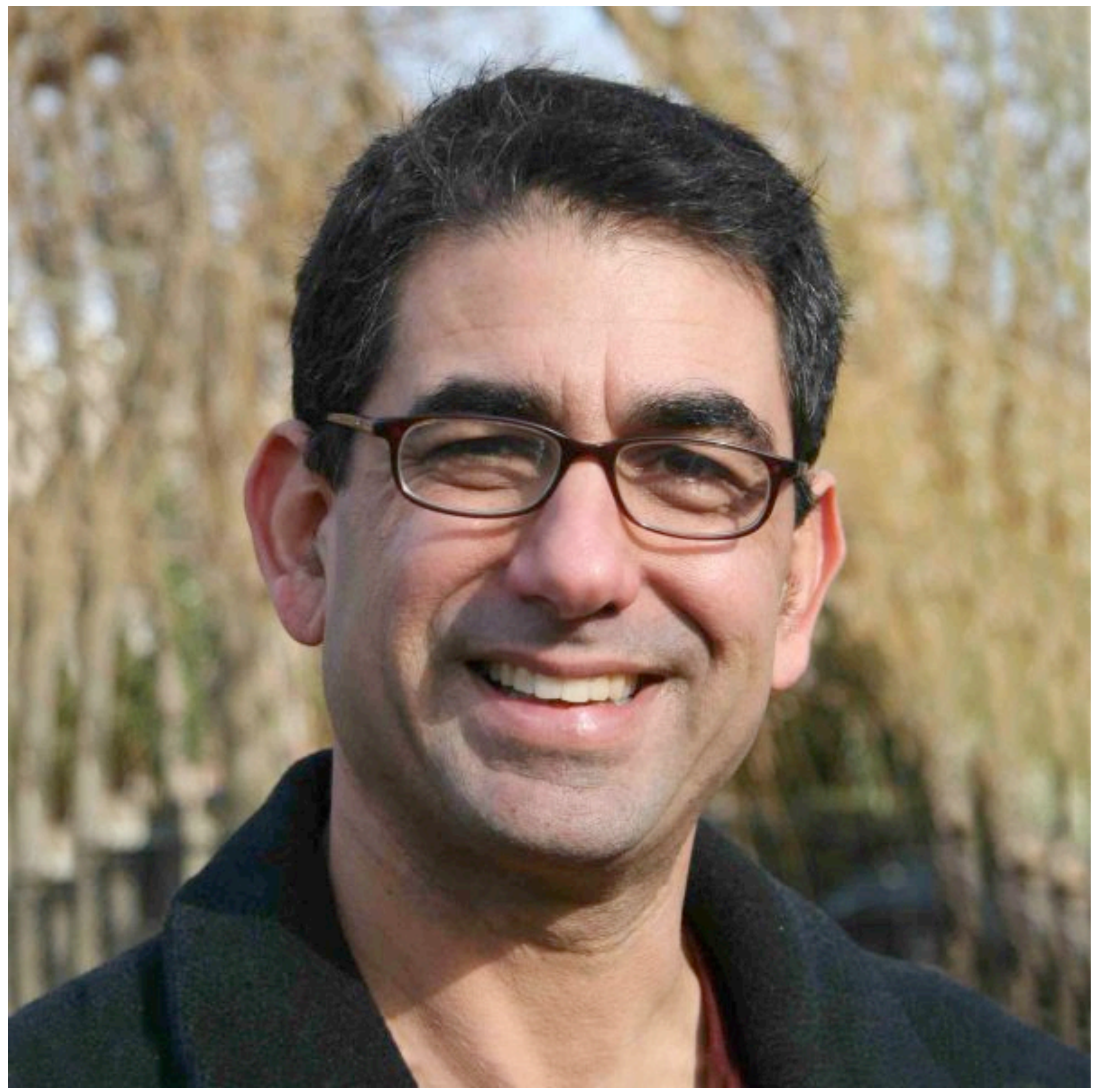

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